## PDC2013 Flagstaff, AZ, USA

## IAA-PDC-13-03-21P

Planetary Defense – Recent Progress & Plans
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## The metric spaces and its applications for NEO Characterization

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Keywords: Metric spaces, Reachability, Relative motion, Control strategy

## ABSTRACT

The methods describing an orbit can be generally divided into two categories, the Cartesian coordinates and the classical orbital elements. The former method naturally belongs to metric spaces, while the metric is difficult to define in the latter space. In this paper, the metric spaces and subspaces of generalized orbital elements are firstly constructed, including the metric of distance, in which the non-negative, symmetry and triangle inequality holds. Then, the compactness and completeness of the metric space and subspaces are deduced. On this basis, a novel intuitive method describing the Keplerian orbits for the NEOs in the spaces of generalized orbital elements is provided.

Secondly, the reachability characteristic described in the generalized orbital spaces is investigated. The effect of impulsive maneuver on the motion of NEO is obvious using Cartesian coordinates. As for the classical orbital elements, the effect can be analyzed employing the Lagrange's planetary equations. In a general sense, the effect can be evaluated comparing the differences of the generalized orbital elements before and after collision, which can be treated as a maneuver. Employing the mappings between the straight line described in the generalized Euclidean space and the impulse characterized in the physical space, the reachable domain for the NEO trajectory generated after single impulse with fixed magnitude and arbitrary direction is studied. The relations between the position and direction of the impulse and the metrics are also obtained based on the theoretical analysis, as well as the numerical simulations. Then a feasible reachability criterion of the NEO is derived.

Thirdly, the relative motion for NEO is modeled in the metric spaces defined previously, and the relative motion for objects that born of the disintegration of the

same parent body is examined based on the reachability analysis. The results can facilitate the NEO characterization, such as the identification of fragments of the same object, finding parent bodies for meteor streams and asteroids that have similar orbits. In addition, the effect of mitigation missions can be evaluated using the criterion developed in this paper.