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Mitigation by Impacts:
Theory, experiments, and code calculations.

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Abstract:

Many diverse methods have been suggested to mitigate the effects of a possible asteroid collision with the Earth. However, interest has mainly focused down to two primary methods. For asteroids less than a few hundred meters in diameter, the direct impact of a multiple-ton spacecraft could change the path of the incoming asteroid so that it would miss the Earth, if that impact could be made a decade or so before the impending impact. For larger asteroids, or later warning, the method of choice has narrowed to the use of a nuclear device detonated near or on the surface of the asteroid. Here we discuss the impact method.

While the impact method appears to be relatively straightforward, important questions remain. Perhaps the most important relates to its efficacy: how much deflection can be achieved with a given impactor? And how does that efficacy depend upon the various parameters of the problem, especially the impact velocity and the asteroid composition? What is the momentum transferred to the asteroid, compared to that of the impacting spacecraft?

This paper will summarize recent results of the authors regarding the theory, laboratory experiments, and numerical calculations of such impacts.

Keywords: *Mitigation, Impacts, Momentum multiplication, Asteroid deflections, Scaling.*

1. Introduction

We consider the use of an impact of a spacecraft into an asteroid to change the asteroid's path so as to make it miss the Earth [1, 2, 3]. There are several serious barriers in our attempts to understand the outcomes of such an impact: 1) they have never been tested or observed; 2) we cannot directly simulate them in the lab because of the experimental size and velocity constraints, 3) we don't actually know the composition of any particular asteroid, and, even if we did, 4) the material responses are so complex that we only crudely model them in numerical code methods. Nonetheless, the literature reports our collective efforts; and, since those efforts are often described as "state of the art" (which they are), they form the basis for our planning.

The kinetic energy approach is simple in principle: impact the asteroid with a large mass (spacecraft). Either run into it with the mass, or have the asteroid run into the mass. With existing or planned lift vehicles we could plausibly deliver ten tons or more of payload to a NEO. An NEO would have a velocity of perhaps 15-20 km/s. That gives possible opportunities for a mission design that would rendezvous with that NEO with a relative impact velocity of several to tens of km/s.

So what is the fidelity of our understanding of such impacts? How can we make progress? We present here our ongoing research effort of theory, numerical simulations, and physical simulations to study these questions.

There is one way to estimate the momentum transfer in impact that relies on existing data. A recent paper (Holsapple and Housen, [6]) uses reported data on the ejecta from impact cratering to estimate that momentum transfer. Since the theory and scaling of impact ejecta is well known (e.g. Housen et al. [7], Housen and Holsapple, [8]) that ejecta theory also provides the basis for the scaling of the momentum transfer problem, and guides the extrapolation of the lab data to the much larger sizes and velocities of interest in asteroid impacts.

But there is also a more direct approach: one can directly measure the momentum transfer without regard to the mechanisms that produce it: by making experiments of impacts into a variety of material types in the lab and, actually measuring the momentum transfer in the impact. We have recently made such experiments and report results here.

And, finally, one can perform numerical code calculations (some prefer to call them simulations) of hypervelocity impacts and monitor the momentum transfer. We also report results of that type.

The organization of this paper is as follows. The underlying theory is given in section 2; section 2.1 includes an outline of the scaling determined by the cratering and ejecta processes, and section 2.2 includes a discussion of the effects of the amount of material that may or may not escape the asteroid's domain. Section 3 includes an outline of our experiments and results: 3.1 contains results using the ejecta data, and 3.2 outlines our recent direct impact experiments. In section 4 we present the code calculations and results. Finally, section 5 has a summary of our conclusions and outstanding issues.

2. The Theory.

The underlying theory has two parts. Since the origin of the momentum transfer enhancement is the ejected material in the impact, the scaling of the momentum results (the dependence on the impact size and velocity) can be obtained by the known scaling for the amounts of ejecta in an impact. But, in an application to a an asteroid, one must also consider the fate of that ejecta. Any ejecta that lands back on the asteroid surface will not provide any net momentum. Furthermore, whatever ejecta is launched at a sufficient speed to escape the asteroid will have a certain velocity and angle "at infinity", and it is that velocity and angle which the determines the net momentum contribution to the asteroid. Following [6], we discuss these two concepts in turn.

2.1 The scaling of the cratering and ejecta

The fundamental basis for the effects of the impact of a payload into an asteroid or comet comes from the law for the balance of momentum. If the impactor has mass *m* and initial velocity *U* relative to an asteroid, then the velocity change Δv of that asteroid with mass *M* is

$$\Delta v = \beta \frac{mU}{M} \tag{1}$$

where $\beta > 1$, is the "momentum multiplication factor". This assumes that the asteroid is hit in any part of its central region, so that the impactor does not richochet off the surface; and that significant material is not broken off from the back surface of the target asteroid. Neither of these is thought to be an issue. Hitting off-center is of no concern, while that might also impart a rotation to the asteroid, it does not change the linear momentum balance.

When the impacting body just buries itself in the target, and no material is thrown out, then the event is "perfectly plastic" and $\beta = 1$. However, a hypervelocity impact into a geological material will usually blast out a crater 10 to 20 times the size of the impactor. As a consequence, the volume of the crater is a few orders of magnitude greater in volume than the impactor. In the cratering process, the material is excavated and thrown out at large velocity, and a substantial component of that velocity is normal to the local surface.

As a consequence, the total impulse imparted to the target body has two parts: the “primary” component from stopping the projectile, with $\beta = 1$, and the “additional” component $\Delta\beta$ from the ejected material, which gives $\beta = 1 + \Delta\beta > 1$. The magnitude of $\Delta\beta$ will be referred to here as the "momentum enhancement".

The scaling theory for hypervelocity impacts and cratering can be used to predict how $\Delta\beta$ will depend on the impact conditions. That theory is based on the fact that the impactor is very small, and transmits its effects very rapidly, in comparison to the subsequent effects such as the crater size and formation time and its ejecta. Therefore the impact can be approximated as a "point source" deposition of energy and momentum. For a point source, the impactor's defining parameters, its radius *a*, velocity *U* and mass density δ, affect the outcomes only according to the magnitude of a single power-law combination $aU^{\mu}\delta^{\nu}$. The applicability of that assumption is well documented by numerous impact and disruption results (Holsapple and Schmidt, [4], Holsapple [5], Housen et al. [7]). The scaling theory predicts that the value of the material-dependent exponent μ must be between 1/3 and 2/3. Many experiments in relatively non-porous materials such as rocks show μ is about 0.55-0.6. For relatively porous materials, experiments give μ to be about 0.4. In all cases the value of the exponent ν has been found to be about 0.4.

Using that point-source assumption, and the assumption that the cratering process is dominated by some strength Y of the asteroid, then the part of the momentum multiplication $\Delta\beta$ that is due to the cratering process can be predicted. The value of $\Delta\beta$ will depend on the strength and the impact velocity as

$$\Delta\beta \sim \left(\frac{\rho U^2}{Y}\right)^{\frac{3\mu-1}{2}} \tag{2}$$

In a competent body, if μ 0.55, then

$$\Delta\beta \sim U^{0.65} \tag{3}$$

so that the change of velocity satisfies a relation like

$$\Delta v \approx \frac{m}{M}\left(U + KU^{1.65}\right) \tag{4}$$

From the data presented below, we find that the constant K in this expression has a value of about 0.5. Therefore, for the same asteroid and impactor mass, the ratio of the velocity increment for an impact at the relative slow speed of 2 km/s and one at the high velocity of 20 km/s would be a factor of 25:1, and not just the obvious ratio of 10 determined by the increase in impactor momentum. Higher impact speeds give increasingly bigger "bang for the buck".

The above discussion applies to impacts on asteroids that are governed by the strength of the surface material. If instead the asteroid were a strengthless "rubble-pile", or sufficiently large, then its surface gravity g would determine the cratering[1]. In any case, in that case the scaling theory predicts that

$$\Delta\beta \sim \left(\frac{U^2}{ga}\right)^{\frac{3\mu-1}{2+\mu}} \tag{5}$$

For a porous material, which is dominated by the surface gravity, expecting that μ=0.4, the velocity increment of the asteroid would be given by the form

$$\Delta v \sim \frac{m}{M}\left[U + K\frac{U^{1.17}}{(ga)^{0.08}}\right] \tag{6}$$

For these materials we find below that the value for K is very low, and the enhancement is small. But, one can expect at least a value of β 1.0.

These formulas allow us to predict how the momentum factor will depend on impact velocity, strength, surface gravity and asteroid size. Those can be used to scale from lab sizes, where experiments are possible, to the actual conditions of interest. This is important because laboratory experiments are small and limited to impact speeds of ~7 km/s, whereas actual deflection missions may occur at velocities several times higher and for much larger bodies.

[1] To be modeled as strengthless, any strength Y must be less than typical lithostatic pressures. That is, a ratio $Y/\rho g d$, where d is a typical crater diameter, must be small. Since lithostatic pressures for 10 m craters on 100 m asteroids are only on the order of a few Pa, a very minute strength might dominate the gravity.

2.2 Escaping material

The scaling above applies to the momentum imparted to an asteroid if all of the ejecta escapes at very high speeds. It may apply to a laboratory experiment in which all ejecta does escape from an experimental apparatus. But for an asteroid, one must also consider the ultimate fate of that ejecta. If a mass element of ejecta is launched, but later falls back to the asteroid's surface, then it provides no net momentum transfer to the asteroid. To consider that effect, an analysis of the paths of mass elements must be considered.

Holsapple and Housen [6], presented such an analysis. As the scaling laws predict, the distribution of ejecta velocities is assumed to be a power-law between a slowest velocity v^* and an upper velocity v_{max}, as shown in figure 1.

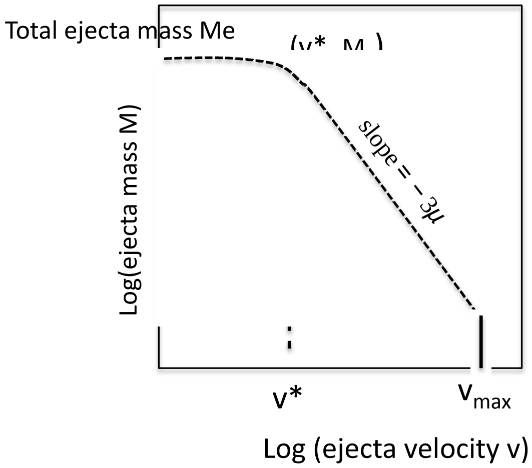

Fig. 1. The assumed cumulative distribution of ejecta mass with a velocity greater than some given velocity. All ejected mass has a velocity greater than the minimum v^*, and the largest velocity is the value v_{max}.

The velocity of a mass element will determine its trajectory. The trajectories of escaping and returning particles are depicted in figure 2.

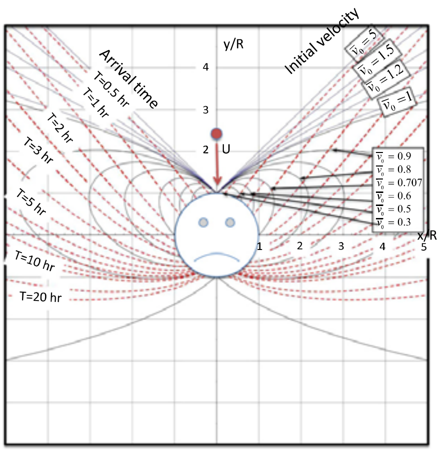

Fig. 2. Black curves: trajectories of escaping or returning material on a plot of coordinates (x/R, y/R) scaled to the asteroid radius R, as determined by the scaled initial particle velocity $\bar{v}_0 = v_o / v_{esc}$. Red curves: the times to reach spatial locations. (From [6]).

So, while there may be ejecta moving as slowly as v*, only that amount traveling faster than the escape velocity v_{esc} of the asteroid will contribute to the momentum enhancement. Since the total effect is obtained by integrating the mass versus velocity distribution, the lower limit of that integration must be v_{esc} and not v^*.

Furthermore, an additional factor must be considered. That material that does escape the asteroid will begin its path with some initial velocity and is typically be launched at a 45° angle to the asteroid's surface; it will then travel through space and ultimately have some velocity and angle "at infinity". It is that final velocity and angle when leaving the gravity field of the asteroid that determines the net final momentum imparted to the asteroid from any ejecta mass element. The total effect for all of the ejected material then follows from an integration over the distribution of velocities. That result is shown here as figure 3, it determines a correction factor F_{esc} that must be applied as a function of the minimum ejecta velocity scaled to the asteroid escape velocity.

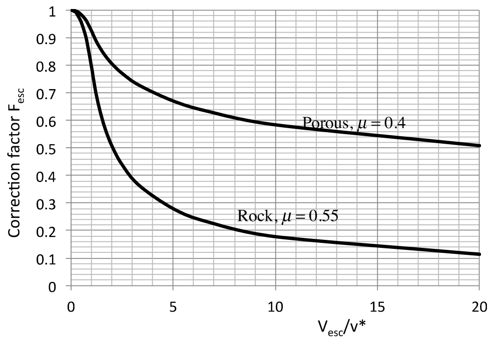

Fig. 3. The reduction factor for the net momentum transfer accounting for the paths of mass elements as they escape depends on the ratio of the escape velocity v_{esc} to the minimum ejecta velocity v*. That factor depends on the scaling coefficient μ, the net result for rocks, and for porous materials is shown. (From [6])

That completes the analysis of the underlying theory. If one has measured the mass-velocity distribution of the ejecta in some experiment, the integration of that distribution gives the momentum transfer for that experiment. The scaling results of Eq. (2) or (5) can be used in application to a different impactor size or velocity. Finally, for in application to a particular asteroid, the corrections for non-escaping mass must be also applied.

We now discuss experimental results.

3. Experiments

As discussed in the introduction, there are two kinds of experiments which can be used to determine momentum transfer. The first relies on the measurement of the ejecta velocity versus mass distribution, and there is a significant amount of such measurements in the literature (Housen et al. [7, 8]).We summarize that approach first. Then we present experiments of the direct measurement of momentum transfer to finite sized bodies in an impact in a laboratory.

3.1. Ejecta measurements

Holsapple and Housen [6] presented a detailed report of momentum enhancement using existing experiments on cratering ejecta. For scaling to asteroid conditions for rocky targets, the above Eq. (2), in which $\Delta\beta$ depends upon the strength Y, was used. However, in addition it was considered that the strength of a rocky body might decreases substantially from laboratory values as asteroid size increases. For granular targets (rubble piles) the Eq. (5) is applied. The net results, using expected values for the scaling coefficients, gave the momentum multiplication $\Delta\beta$ formulaes shown in the following Table 1.

Asteroid Composition	Formula for momentum multiplication
Lab strength strong rock	$\beta = 1 + (0.13) U^{0.65} Y^{-0.325} \rho^{0.125} \delta^{0.2} F_{esc}$
Asteroid Strong Rock, strength scaled from lab values (assumes a representative crater size from the impactor).	$\beta = 1 + \left(6.5\ 10^{-4}\right) U^{0.65} \rho^{0.125} \delta^{0.2} F_{esc}$
Granular (Dry sand, porosity ~30%, friction angle 35˚)	$\beta = 1 + 0.042 U^{0.23} (ga)^{-0.11} \delta^{0.2} \rho^{-0.2} F_{esc}$
Granular (Loose sand, porosity ~20%, friction angle 25˚)	$\beta = 1 + 0.07 U^{0.23} (ga)^{-0.11} \delta^{0.2} \rho^{-0.2} F_{esc}$

Table 1. Results from Holsapple and Housen [6], which used ejecta measurements to calculate the momentum multiplication. Here U is the impact velocity Y is the target's strength, ρ is the target mass density and δ is the impactor mass density. The term F_{esc} is the correction which must be applied for the non-escaping ejecta and is a function of the asteroid size. See [6] for the details.

In [6], these results were applied to a representative cases considering two assumed impact velocites into a 500 m asteroid of varoius compositions. The reader can refer to [6] for those results.

3.2. Direct momentum measurements

While the use of ejecta data can be used to determine beta, that method is not highly accurate. It depends in a very strong way on the assumed minimum ejecta velocity *v**, which is hard to measure in the experiments. An alternative method uses the direct measurement of the momentum imparted to finite targets in a laboratory setting. Housen and Holsapple [9, 10] reported such experiments, they were conducted at the Boeing impact facility and at the NASA Ames Vertical Gun Range. The target materials included dry sand and two natural rock types: one was non-porous and one was highly porous. The impacts occurred vertically and normal to the target surface, and the impact velocity ranged from 0.5 to 5.7 km/s. The primary goal was to measure the β momentum multiplication factor.

To do so, the target was suspended by springs, and impacted vertically. The resulting vertical oscillation magnitude allowed the momentum transfer to be determined. In those cases where several modes were present in the record, the application of a FFT was used to assess the magnitude of the primary one. High speed cameras recorded the oscillations as well as the cratering and/or disruption processes. The experimental apparatus is shown in figure 4:

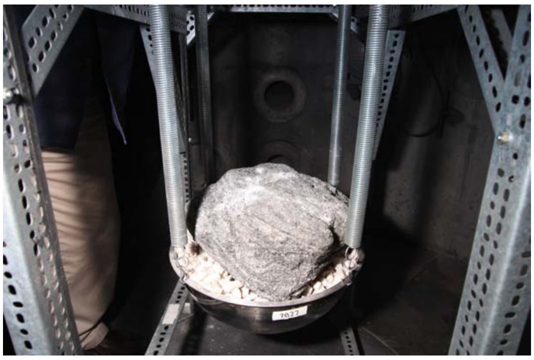

Fig. 4. A rock target suspended on springs. The projectile travels vertically to strike the top of the sample. The magnitude of the resulting vertical oscillation is directly proportional to the momentum imparted to the target.

We haver performed about 20 such experiments. The following figure 5 shows our results, in addition to those of others.

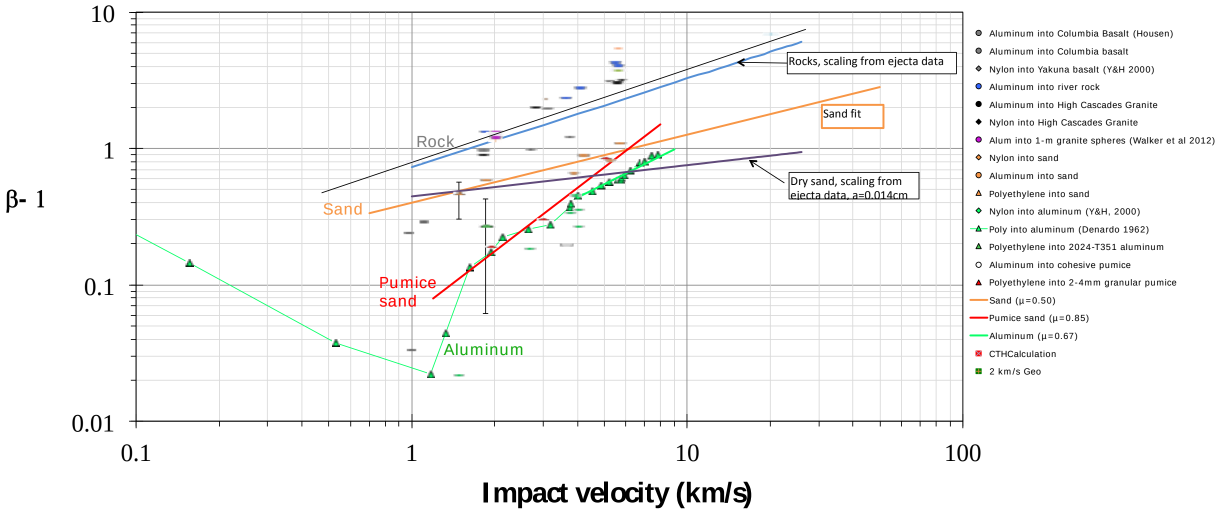

Fig. 5. Many results for the momentum enhancement β as a function of impact velocity. The values towards the bottom indicate very little momentum enhancement while the values near the top represent very significant momentum enhancement. The most significant enhancements are for the hard rock targets. Actual experimental data in that case increase with velocity to a maximum measured $\beta=4$ at the velocity of about 6 km/s. Those values are corroborated in two ways. The blue line represents the predictions based upon the ejecta measurements and the scaling theory, and compare very well. The four red squares represent numerical calculations as described in the next section. Those confirm the scaling predictions to the higher velocities. The values for the porous sand targets are substantially lower, and the the increase with velocity, as predicted by the scaling theory, is less.

For the rock targets with the same strength as the laboratory samples, these momentum enhancement values can be used directly, and little or no correction for non-escaping ejecta is needed, because all of the ejecta is at high speed. But, if one accepts that the target strength decreases with target size D as $Y \sim D^{-0.5}$, an additional factor is indicated from the scaling. In that case, using Eq. (2), μ=0.55, and scaling from a 10 cm lab sample to a 200 m asteroid, an additional factor of $(2e4/10)^{0.325/2}$ =3.4 can be applied. If that is indeed true, then at 20 km/s it is predicted that β*=24! Hot diggidty…*

For porous targets, these results depend upon the gravity and impactor size, according to the Eq. (5). Note that the product *ga*~160 $(cm/s)^2$ in these lab experiments. However, a one meter impactor hitting a 200 m asteroid, with surface gravity only about 0.01 cm/s^2 would have *ga* ~100, so that the term $(ga)^{-.083}$ is essentially the same in these two cases. That is, out 1G experiments at small scale are very close to a correct simulation of that actual asteroid case with a large impactor and small surface gravity[2]. However, in those cases the correction accounting for the amount of ejecta escaping will reduce that enhancement. Specific numerical examples are presented in [6].

4. **Code Simulations**

We have also run computer code calculations of hypervelocity impacts into finite targets. We use the Sandia CTH wave code, which is well documented elsewhere. The most interesting, and challenging, calculations are for the rock targets where the momentum enhancements are the most significant. We have to date focused on those.

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2 The non-dimensional term in Eq. (5) is essentially a Froude number.

It is well known that a major uncertainty of such code calculations relates to the models for the complex material behavior under the wide range of pressures and temperatures encountered in a hypervelocity impact [13]. Our calculations included four specific features of material behavior. We use equations of state from the Los Alamos Sesame library for the various materials, which relate the pressure, density and temperature, and include melt and vapor phase changes. We use a porosity model [12, 13] which is similar to, but an extension of the well-known P-α model of Herrmann [14]. We use a shear-strength model, developed by the lead author, that includes a damage parameter, and a pressure, strain rate and temperature dependence. Finally CTH has a model for spall tensile failures that introduces voids and relaxes the stress in any cell when the maximum tensile strength exceeds some prescribed value given as an input.

There are many uncertainties and parameters in these models. For that reason, numerical calculations cannot be assumed to be correct unless, at the least, they are tuned to, and compared to, known experimental results. Our experiments are dominated by the spall behavior of the material. Therefore, to validate our models we tuned the calculations to match a detailed experiment of an impact into a basalt rock cylinder, which was also dominated by spall. That experiment was of a 2 km/s impact into a basalt cylindrical target [11]. The final results of the experiment are shown on the left of figure 6 while the numerical code results are shown on the right. It is seen that the match is excellent.

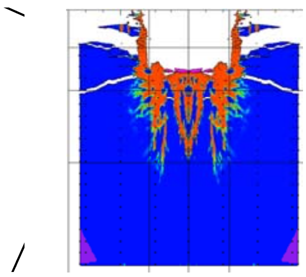

Fig. 6. A code fidelity-assessment exercise. The picture on the left is the outcome of a 2 km/s impact into a granite cylinder. The picture on the right is a CTH code calculation of that experiment, at a time before the ejecta has departed, but after the shock waves have disupted the material. The dominant features in both cases are the tensile spall fractures, indicating that the modeling of the material must be most accurate in dealing with tensile fractures. The final match is excellent.

Next, calculations were made of impacts into finite spherical targets at various velocities. A typical one used a 0.3 cm diameter aluminum projectile impacting a 20 cm spherical rock at 5.6 km/s. The calculational cells were only 0.1mm, resulting in 64 of them across the diameter of the projectile, as depicted in figure 7.

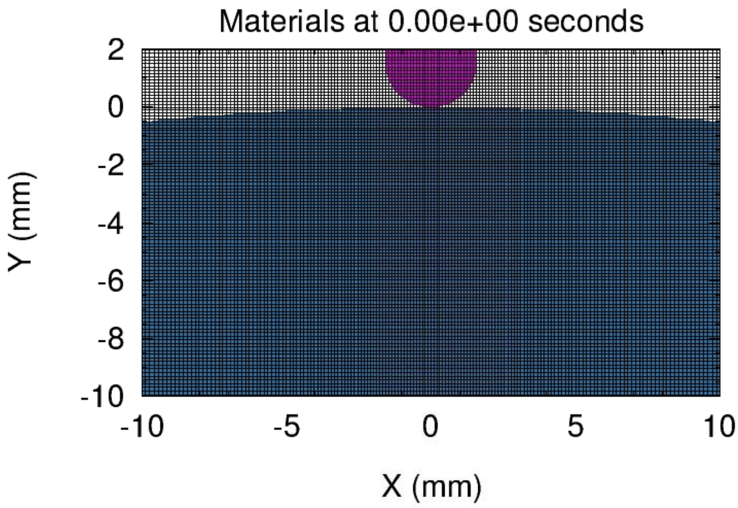

Fig. 7. The initial setup for the calculations. The cells at the center are of 0.1 mm in size, giving 32 across the 0.32 cm diameter projectile, and about 1000 across the target diameter. While this calculation was run in 2D, an equivalent 3D calculation would require almost 10^9 cells. The calculation was completed in about one day on an 8-core Mac desktop computer.

A final calculated outcome of that calculation is presented in figure 8. A flat spall crater about 7 cm diameter and 1 cm deep was formed. In addition, while it is hard to see in this small-scale figure, tensile cracks were formed to a depth of about 5 cm.

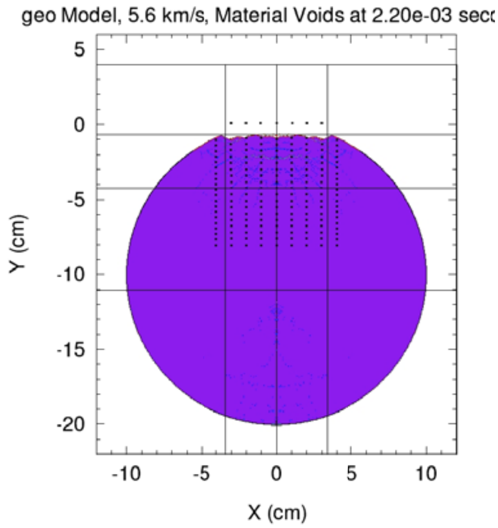

Fig. 8. The final crater obtained in the numerical calculation. It is clearly dominated by near-surface spall. Numerous tensile cracks were obtained, especially within 5 cm of the crater floor. The dots indicate marker particles used to plot outcomes. There are numerous tensile cracks beneath the crater floor.

The picture of the actual experimental crater obtained in a rock target at 5.6 km/s is shown in figure 9. The agreement of the calculation with the actual event is excellent. The most obvious discrepancy above in figure 8 is the failure to model the rock as a flattened sphere.

Fig. 9. The crater from the experiment at 5.6 km/s. The diameter, while irregular, is about 7 cm, and the depth is just less than 1 cm.

Calculations were made for four different impact velocities ranging from 0.5 to 20 km/s. The results for the momentum enhancement were indicated previously on the figure 4 plot above. Not only do these calculations match the experimental points, they provide confidence for the extension of the experimental results to the higher 20 km/s impact velocity, and, in addition, further validate the scaling theory.

5. Summary.

Great progress has been made in understanding the momentum enhancement that can be expected in a hypervelocity impact into an asteroid. The scaling theory has been formulated, and it indicates the important role of the impact velocity, over and above its role in determining the momentum of an impactor. Many existing experiments on ejecta from impact craters exist, and have been mined to determine magnitudes of momentum enhancement. In addition, we have now performed laboratory experiments directly measuring the momentum transmitted to targets of a variety of material types. Finally, we have made computer code calculations for impacts into rock targets and obtained excellent agreement with the theory and laboratory experiments.

The results clearly indicate the dominant role of porosity in the response of an asteroid target. The momentum enhancement in a rock target may be a factor up to as much as 20, while in a highly porous target there may be no enhancement whatsoever.

Obviously there remain many unanswered questions. We must develop a better understanding of exactly what target properties determine the outcome. We must develop a better understanding of the actual bodies which may threaten the Earth. What if the body is an M type? What if it is a comet? What is the role of a surface regolith on the impact process? What about a porous target with large blocks? How are the blocks formed, and will they be cast away from the asteroid, perhaps enhancing the momentum transfer? Does a 200m "rubble-pile" asteroid really have zero strength? What is its strength? What about highly porous rubble-piles, does the gravity scaling really apply? How hard can we hit an asteroid without disrupting it? Is that bad? What role does an asteroid's spin have on its disruption? Can we perform experiments at low g to directly measure the dependence that the scaling theory indicates on the surface gravity? Can we improve our numerical calculations? Might we spall off material from the antipode of a target, thereby reducing the momentum transfer?

There are also many important questions regarding mission design. For example, what is the likelihood of actually hitting an asteroid of, say, only 100 m diameter at a speed in excess of 20 km/s? What if we hit it at an angle, or in the wings? Much remains to be discovered.

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