Uncertainty Quantification in Impulsive Deflection Scenarios

PDC 2013

April 15-19, 2013

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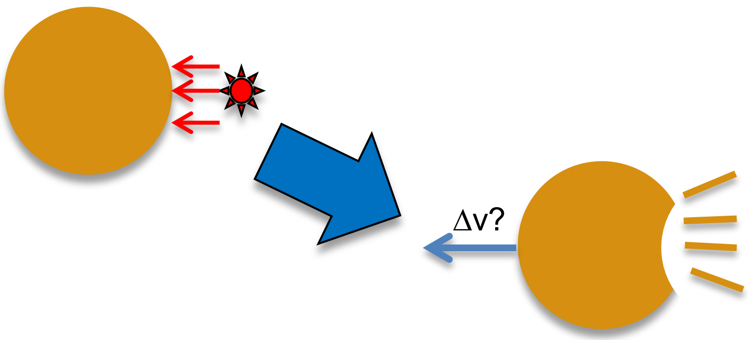
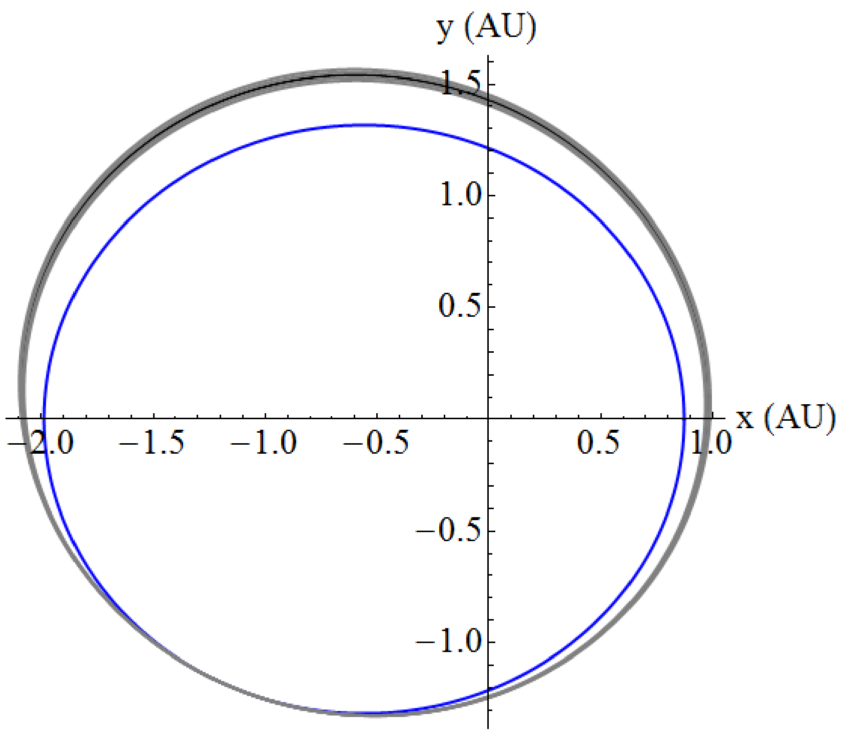
LLNL-PRES-632233

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344, and partially funded by the Laboratory Directed Research and Development Program at LLNL under tracking code 12-ERD-005.

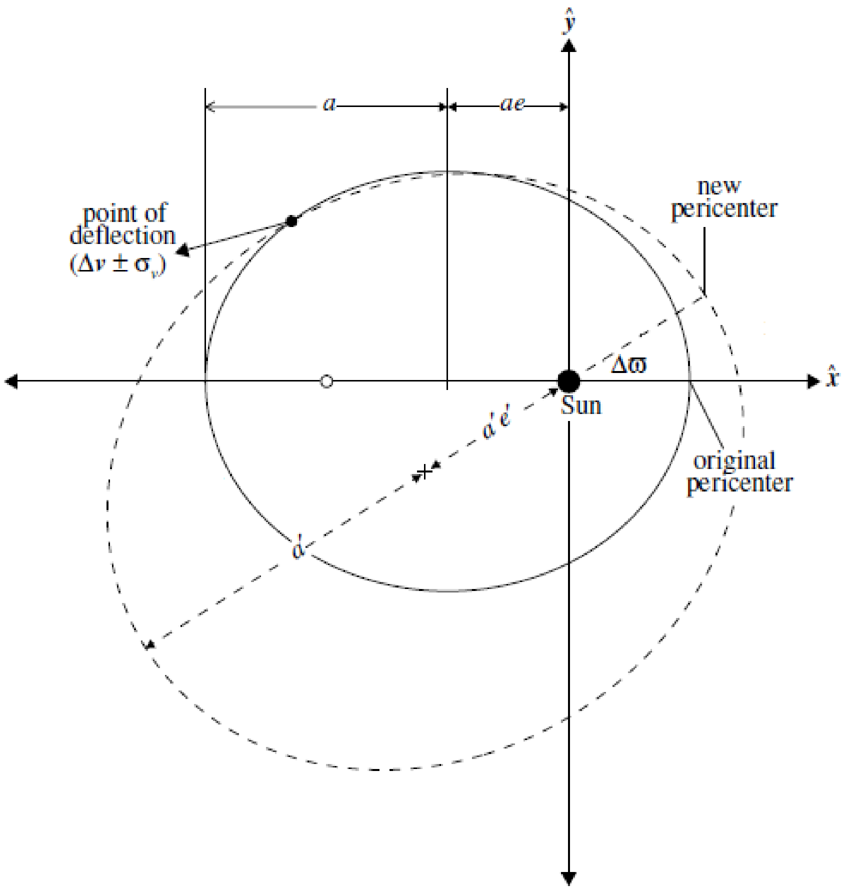
Δv & Uncertainty

- Impulsive Δv
  - Modify Orbit Params. a, e, ω
  - Modification propagates through orbital motion
- Uncertainty Sources
  - Object Composition
  - Impulse Coupling (Nuclear Standoff/Surface/Subsurface, Massive Impactor)
  - Original Orbit Parameters
- Implies $\sigma_{\Delta v}$
  - Propagation through orbit & effect on deflection scenario?





Problem Setup – Orbital Parameters Change

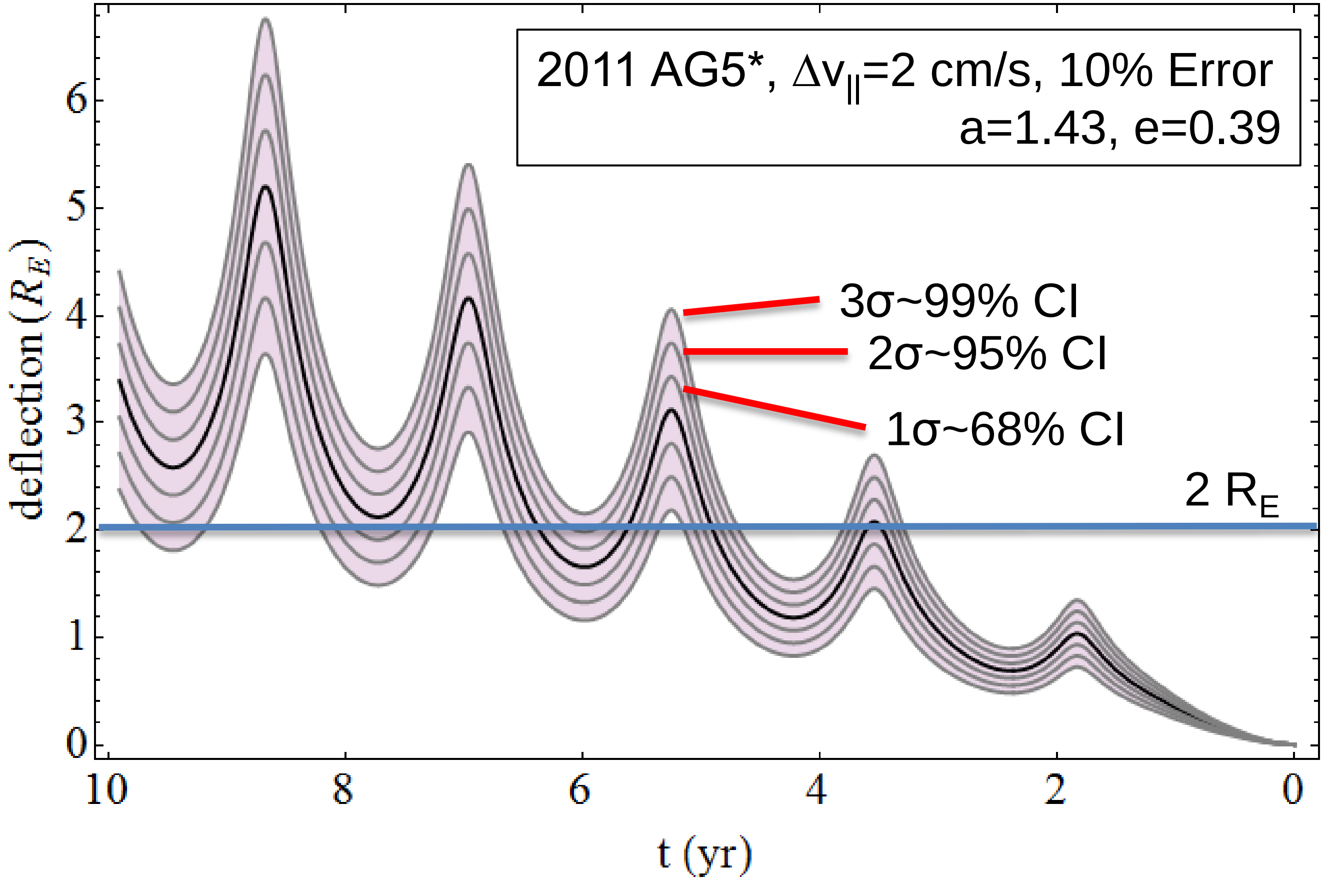


- Go from Δv to Δ_{orbit}(t)
  - Neglecting Earth gravity
- Change to Specific E/H.
  - $U = -\frac{\mu}{2a}, H = \sqrt{\mu a(1-e^2)}$
- Solve for new parameters:
  - a', e', ω'
- Build into new orbit
- Determine time to point of impact versus point of deflection
- Deflection Distance
  - $\Delta r(t) = \sqrt{(x'(t) - x(t))^2 + (y'(t) - y(t))^2}$
- Highly Nonlinear In: Point of Deflection, Initial Orbital Parameters, Δv
- Propagate error through full equations

Error Propagation

- Assume Gaussian error in Δv
  - Not strictly necessary, but typical
- Deflection Error then defined as
  - $\sigma_{\Delta r}{}^2 = \sigma_{\Delta v_{||}}{}^2 \left(\frac{\partial(\Delta r)}{\partial \Delta v_{||}}\right)^2 + \sigma_{\Delta v_{+}}{}^2 \left(\frac{\partial(\Delta r)}{\partial \Delta v_{+}}\right)^2$
- Deflection magnitude error usually dominates:
  - $\sigma_{\Delta r}{}^2 = \sigma_{\Delta v}{}^2 \left(\frac{\partial(\Delta r)}{\partial \Delta v}\right)^2$
- Calculation of this derivative is straightforward
  - …but mind-numbingly long and boring, so I'll spare you the details

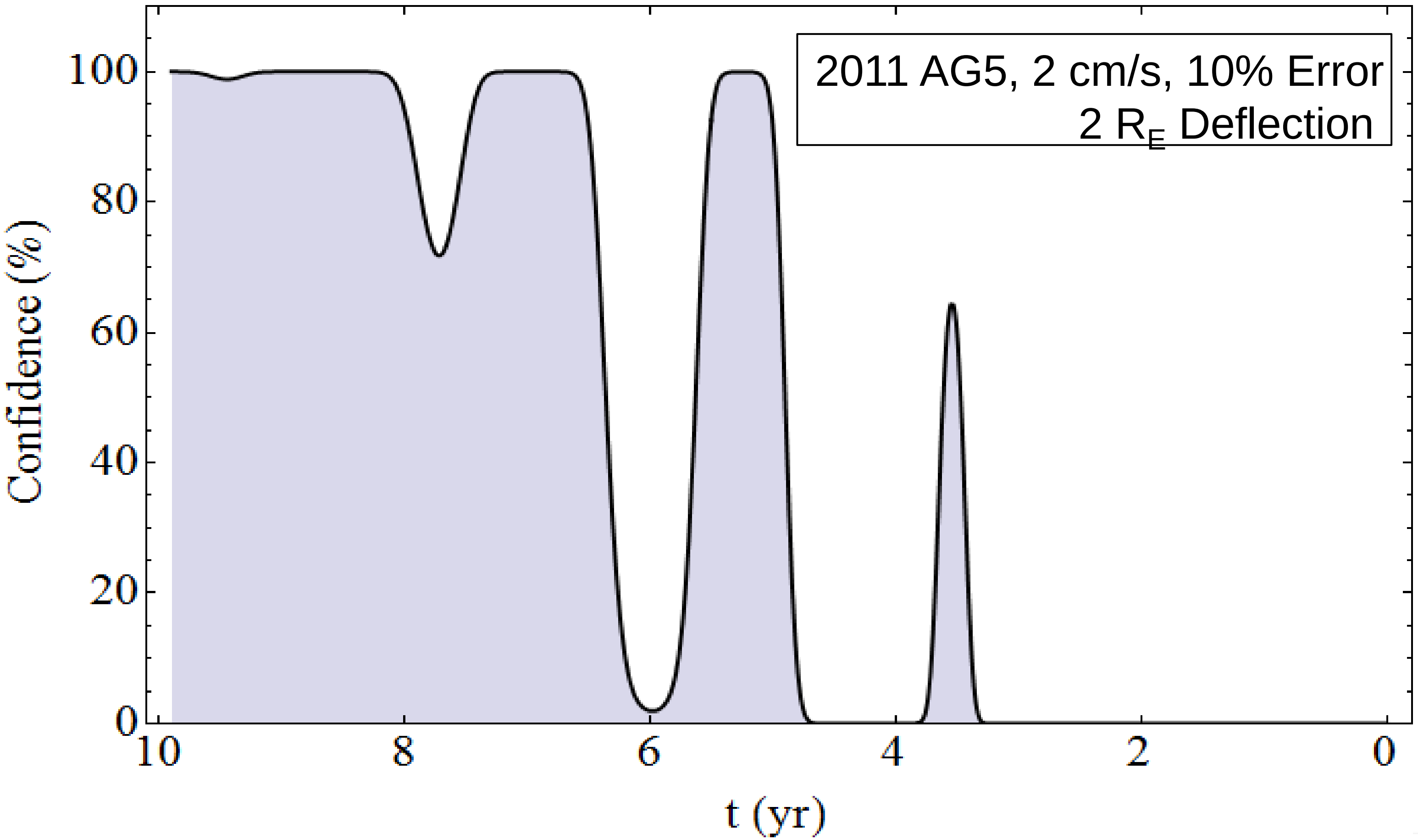
Deflection Errors – 2011 AG5*



- Deflection distance (in Earth radii) vs time from deflection to impact for 2011 AG5-like object
- Deflection error dominated by term linear in Δv
- What confidence level corresponds to a 2R_E deflection?

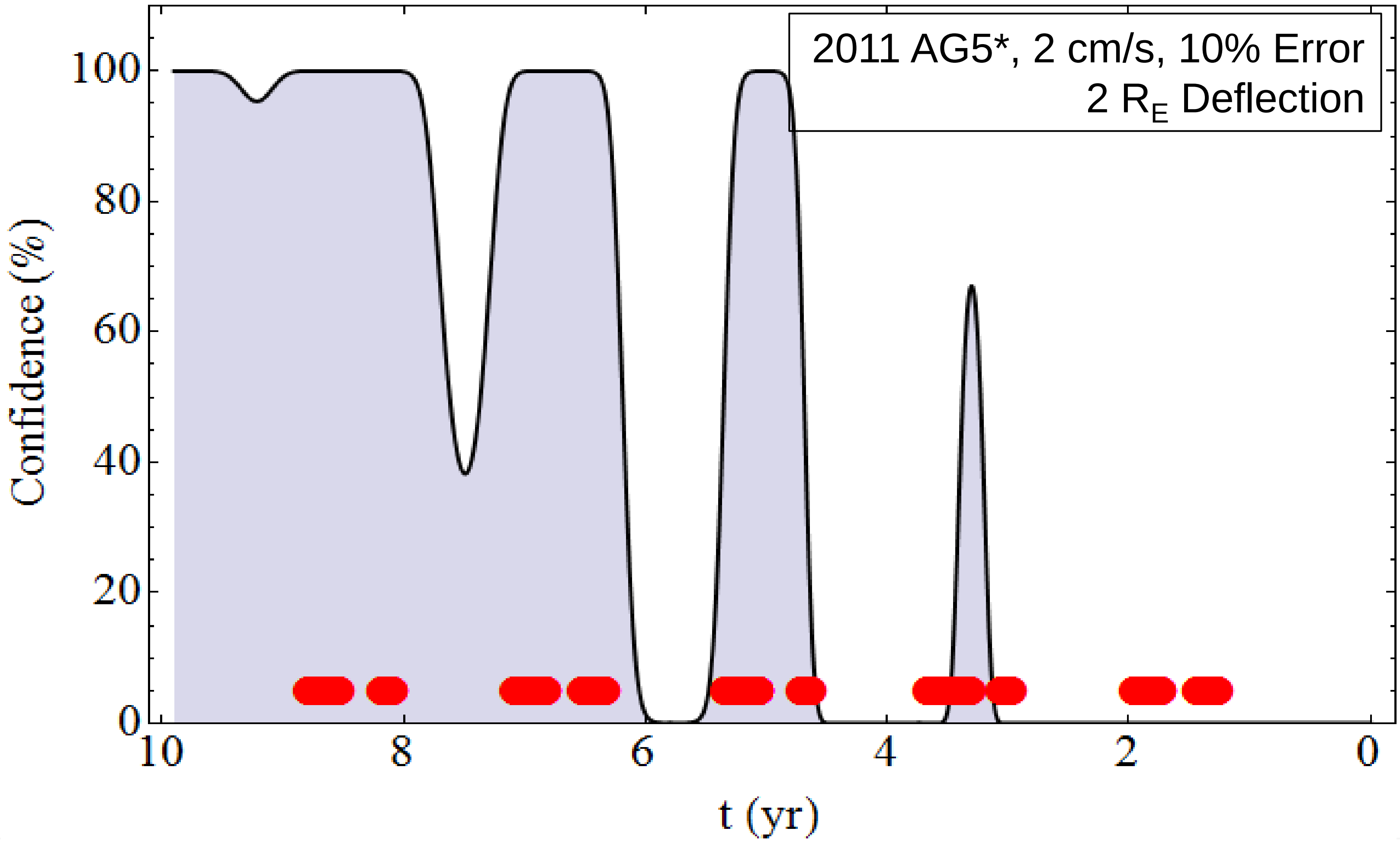
Deflection Confidence

- Percent probability that asteroid will miss by $2R_E$ or more vs time to impact
- Fold with launch window info for mission success probability….



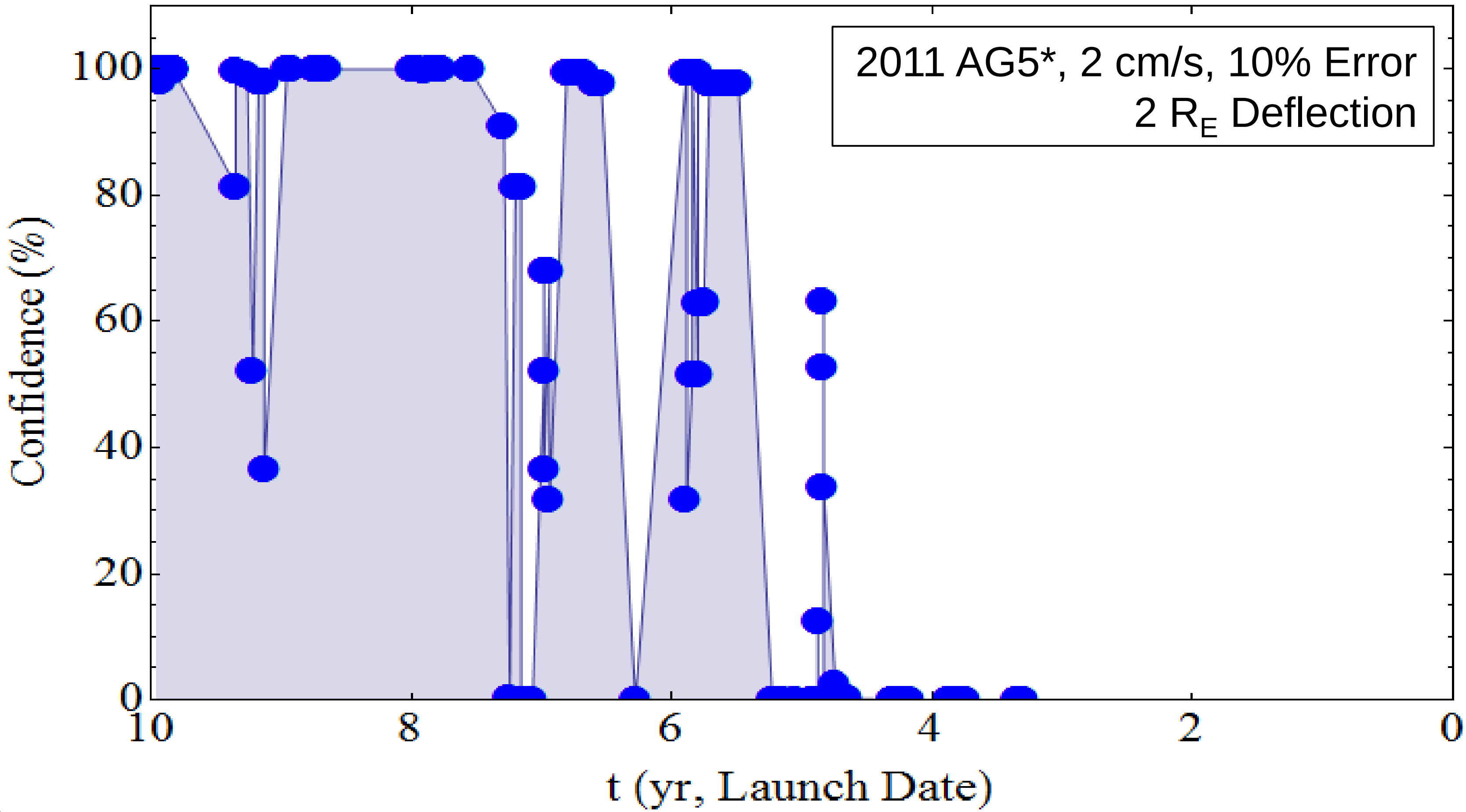
Deflection Intersection Opportunities

- Launch intersection points, using NASA's Trajectory Browser
- Assumes Feb. 4, 2040 Impact

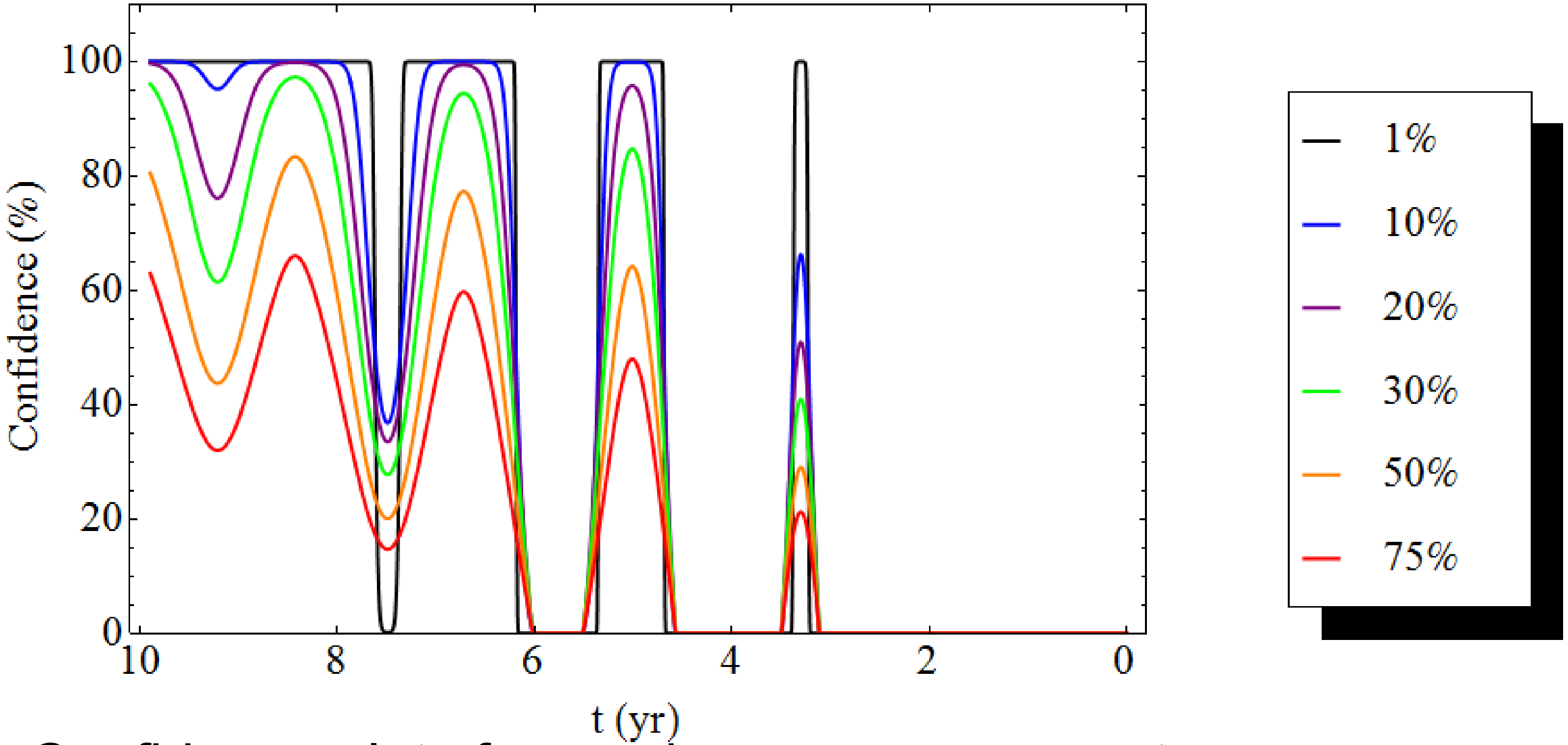


Deflection Launch Opportunities

- Launch Dates & Confidence for 100 Trajectories
- All “Last Chance” Launches Occur 1764 days out: Apr. 7, 2035



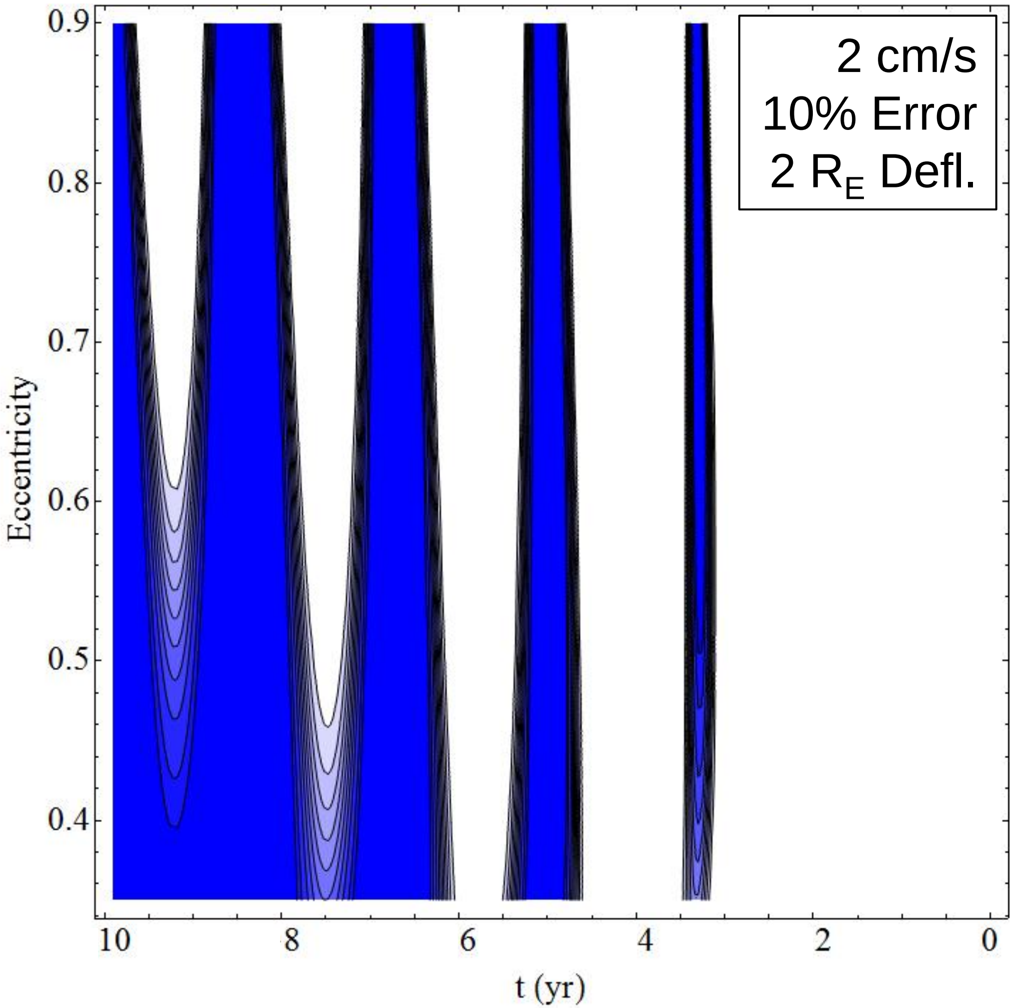
Confidence Suppression



- Confidence plots for varying Δv error percentages
  - Normalized distributions with high deflection velocity tail limited to 20%
- As Δv error increases, overall deflection probability decreases
- Especially noticeable in ‘last chance’ window

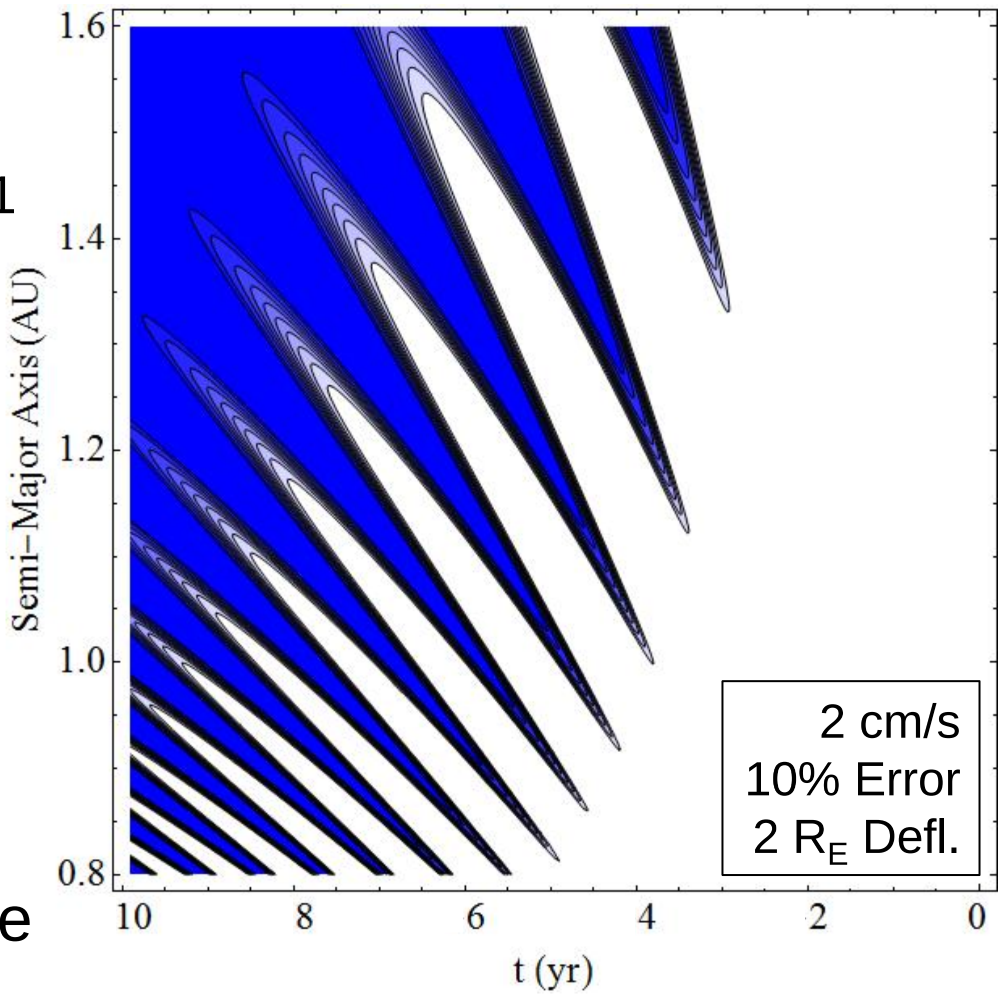
Confidence - Eccentricity Dependence

- Contour plot of confidence levels every 10%, also at 95%
  - Bluer is higher confidence
  - Using 2011 AG5's semi-major axis
- Higher Eccentricity Narrows Response Windows



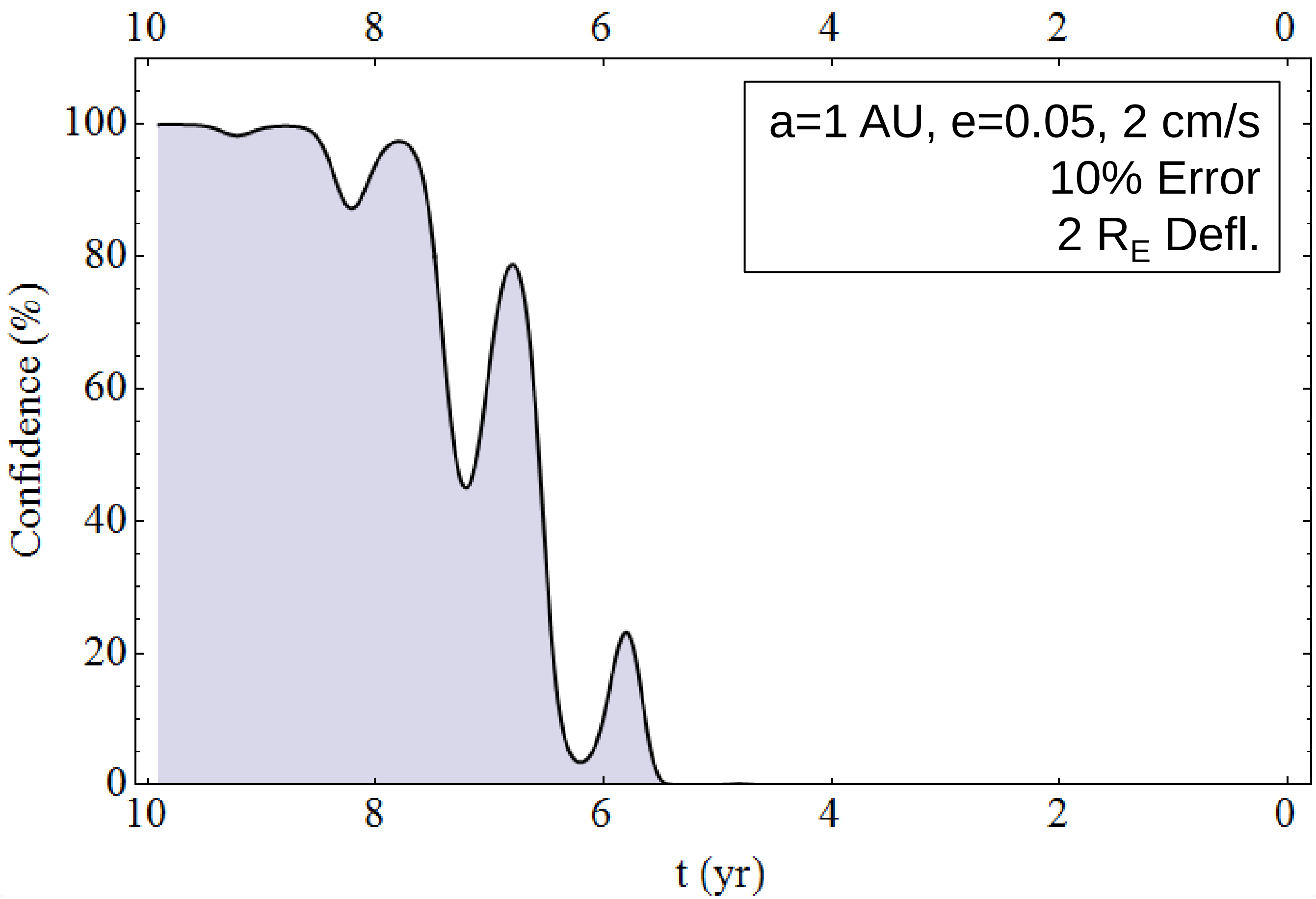
Confidence – Semi-major Axis Dependence

- Contour plot of confidence levels
  - Bluer is higher confidence, 2011 AG5's eccentricity
- Structure largely reflects orbital period differences
- More chances, but smaller windows for small values
- Fewer chances, but larger windows for large values
- Window timing very sensitive to semi-major axis value



Deflection Confidence – Earthlike Orbit

- Peaks lower, troughs higher
- "Last Chance" Window Earlier

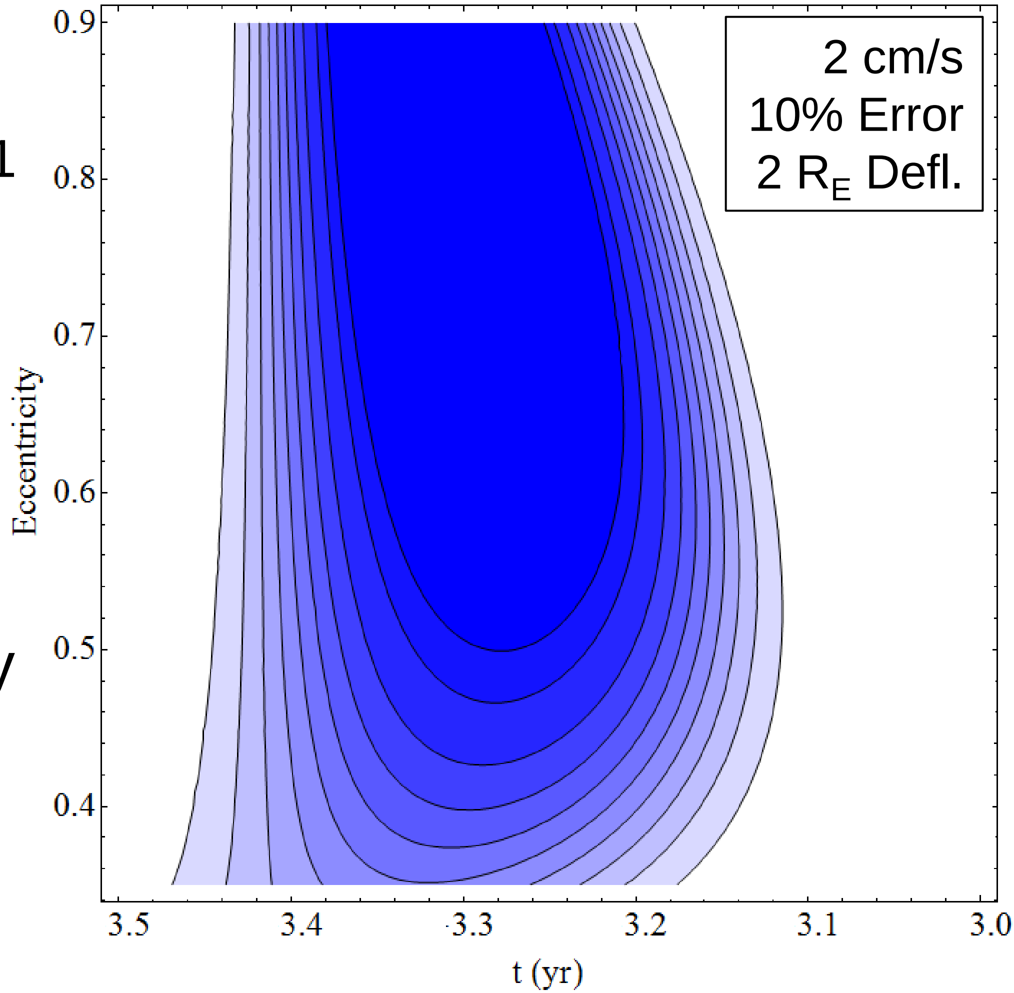


Conclusions

- Uncertainties in object composition, impulse coupling, & orbital parameters can be significant
- Final deflection error dominated by term linear in Δv
- For deflection of 2011 AG5-like object by 2 cm/s:
  - High success probability windows form around orbit perihelion
  - Greater Δv uncertainty broadens windows, but lowers overall success probability
  - Higher eccentricity narrows response windows & changes timing for the “last chance” window
  - Higher semi-major axis gives fewer (but broader) windows
  - Window timing very sensitive to semi-major axis
- For earth-like orbit, higher confidence, but must act earlier

Confidence - Eccentricity Dependence

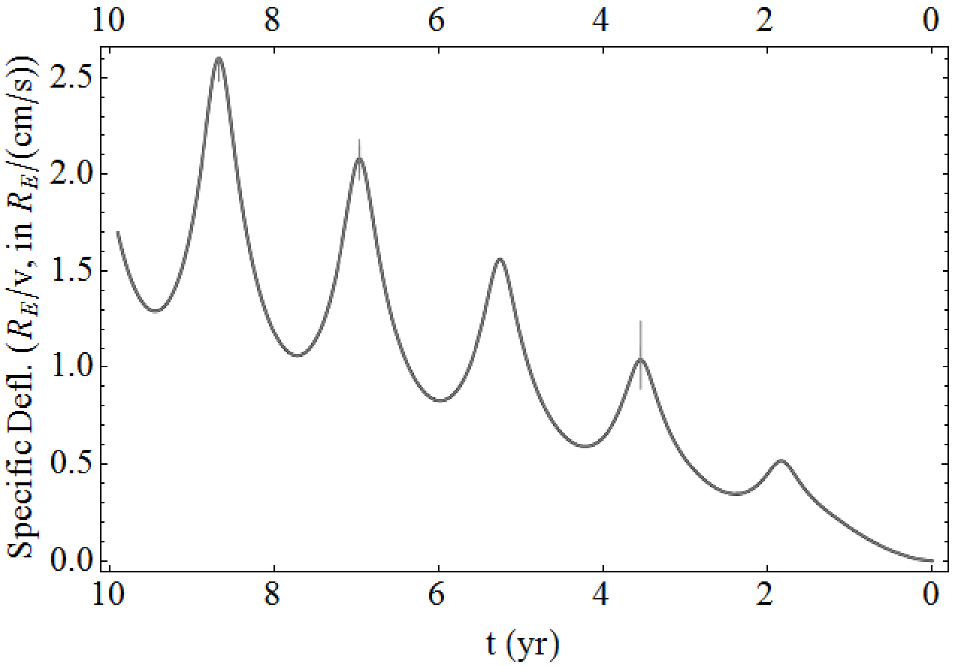
- Detailed contour plot of confidence levels
  - Bluer is higher confidence, 2011 AG5's semi-major axis
- High confidence lines dipping down, not pushing up
- Due to "Last Chance" window being fundamentally different
- Note timing changes for different eccentricities



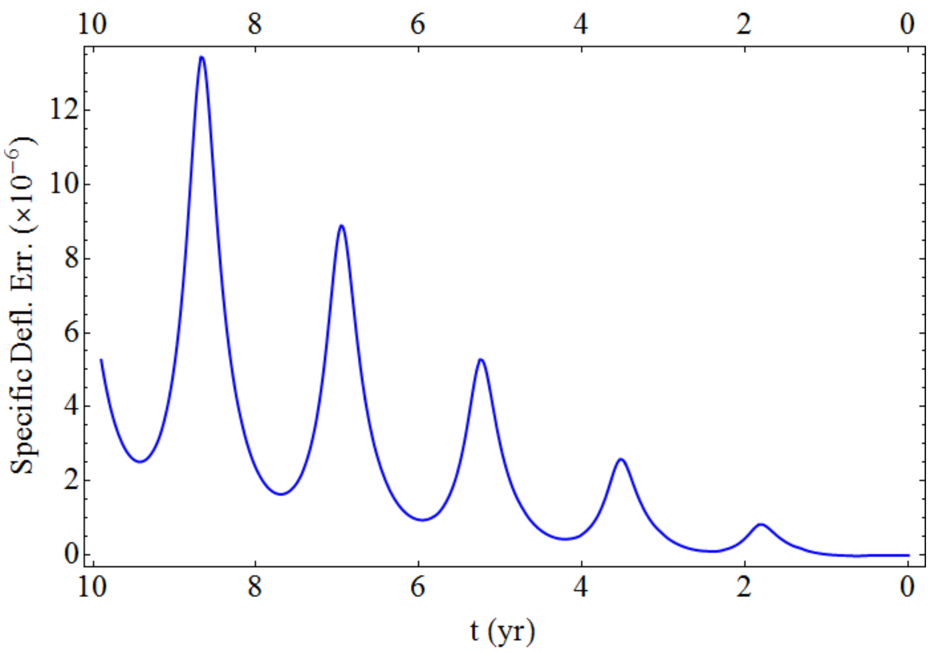
Characterization of Δv Error Nonlinearities

- Specific Deflection: Δr/Δv
  - Majority of error hidden in division by Δv
  - Nonlinearity peaks at perihelion, but still small

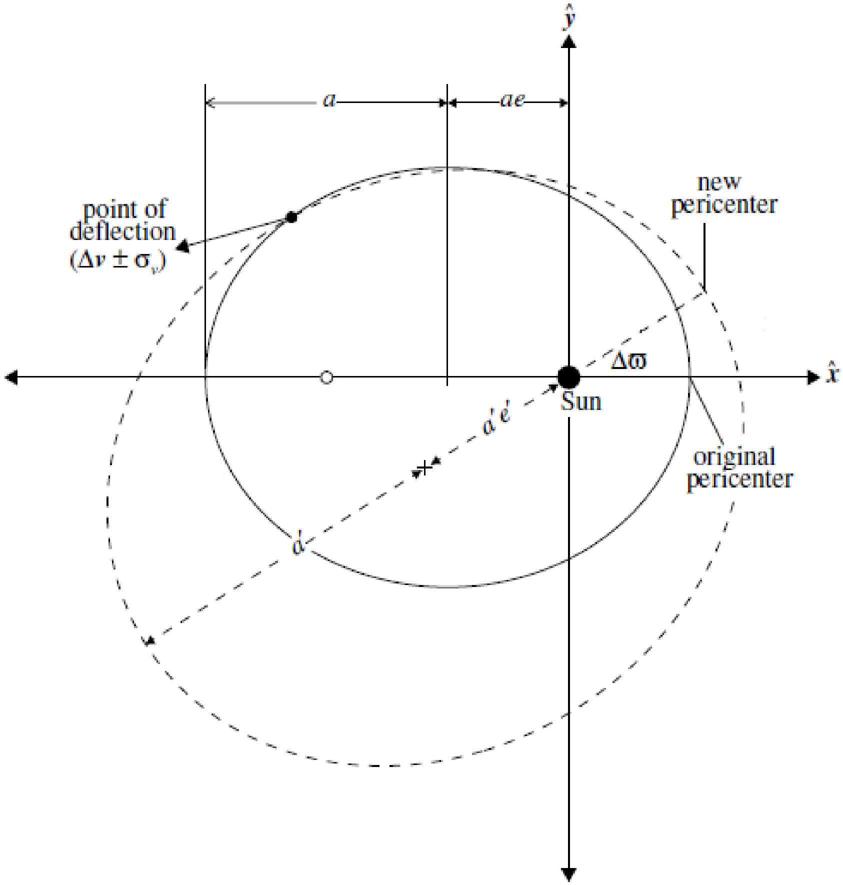
Deflection Per Unit Deflection Velocity



Nonlinearity Residual Error

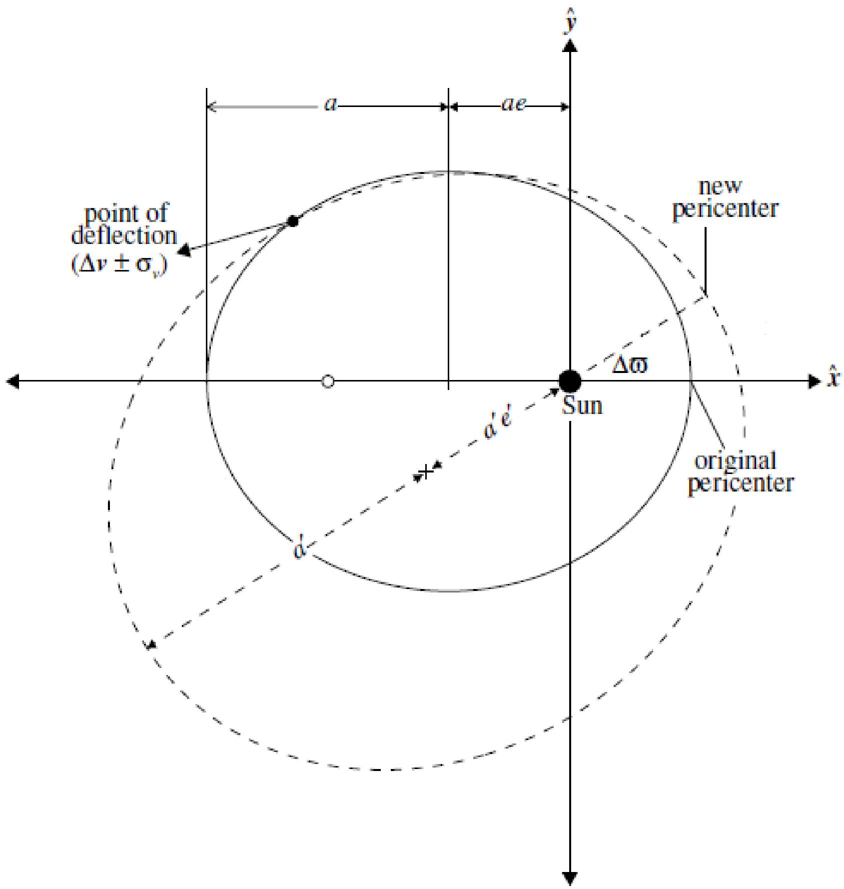


Problem Setup – Orbital Parameters Change



- Go from Δv to Δ_{orbit}(t)
  - Neglecting Earth gravity
- Change to Specific E/H:
  - $\Delta U = \frac{1}{2}\left(\Delta v_r{}^2 + \Delta v_\theta{}^2\right)$ $+ \sqrt{\frac{\mu}{a(1-e^2)}}\left(\Delta v_r e \sin f_0 + \Delta v_\theta (1 + e\cos f_0)\right)$
  - $\Delta H = \frac{a(1-e^2)}{1+e\cos f_0}\Delta v_\theta$
- Solve for new parameters:
  - $U = -\frac{\mu}{2a}$, $H = \sqrt{\mu a(1-e^2)}$
  - $a' = a + \Delta a = a + \frac{\mu \Delta U}{2U(U+\Delta U)}$
  - $e' = e + \Delta e = \sqrt{1 - \frac{(1-e^2)(H+\Delta H)^2(U+\Delta U)}{UH^2}}$
  - $\omega' = \Delta\omega = f_0 - \cos^{-1}\left(\frac{a'\left(1-e'^2\right)}{a(1-e^2)}\frac{1+e\cos f_0}{e'} - \frac{1}{e'}\right)$
  - Timing Considerations…

Problem Setup – Orbital Difference



- Build into new orbit
- Determine time to point of impact versus point of deflection
  - Assume circular 1 AU Earth orbit
- Deflection Distance
  - $\Delta r(t) = \sqrt{(x'(t) - x(t))^2 + (y'(t) - y(t))^2}$
  - X and Y info important for specific positioning at closest approach, but not discussed here
- Highly Nonlinear In:
  - Point of Deflection
  - Initial Orbital Parameters
  - Δv
- Propagate error through full equations