ON THE KEYHOLE POSITIONS OF APOPHIS

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ABSTRACT

The structure of initial conditions, corresponding to possible collisions of asteroid Apophis with the Earth, is very complicated (Sokolov et al., Solar System Research 2013, V. 47, N 5, P. 441-447). This structure is similar to fractal structure due to resonant returns. We discuss the case of asteroid Apophis, because its possible collisions have been carefully investigated and Apophis will be hazardous as before despite orbit refinement in 2013. Other hazardous asteroids have similar complicated structure of keyholes.

The method of approaches and collisions detection includes initial conditions variation using Everhart integrator and ephemerides DE405. We change only one variable (semi-major axis). Initial conditions variation in the future (01.05.2035 for Apophis) used as well in order to overcome the loss of accuracy in the case of encounter in 2029. Important characteristics of investigated trajectories, including relative positions and sizes of keyholes, leads to collisions, are stable under small changes of motion model (Sokolov et al., Solar System Research 2012, V. 46, N 4, P. 291-300). The calculations are performed using a computer claster of the Saint-Petersburg State University.

We derive and discuss the list of possible collisions of Apophis in XXI century. It contains very many keyholes, despite Apophis orbit refinement in 2013. Only essential keyholes are presentrd now in the NASA website http: / / neo.jpl.nasa.gov /risk. In addition to 9 possible collisions in XXI century in this website, important keyholes corresponds to collisions in 2055, 2056, 2064, 2066, 2068, 2068, 2074, 2075, 2078, 2087, 2098. Only in the region of initial semi-major axes between collisions in 2060 and in 2076 (in the NASA website) located more than 100 keyholes.

To estimate sizes of the keyholes, the range of semi-major axes δa at 01.05.2035, corresponding to each collision, was calculated. Alternative method of estimation – using the range of minimum geocentric distances Δr_{min} at 2051 for trajectories, corresponding to each collision.

To avoid a collision, we must move Apophis to the region of semi-major axes without keyholes (Yeomans D.K. et al., 2009 IAA Planetary Defense Conference, Granada, Spain, 2009). We investigate the time dependence of semi-major axis regions, leads to collisions, and evolution with time the regions without collisions. Sizes of regions, leads to collisions, as well as regions without collisions, increase with time. Sizes of regions of possible asteroid motion decrease with time due to the orbit refinement. After 2029 dimensions of regions without collisions, as well as regions leads to collisions, should be multiplied on about 10^5. The next significant change of dimensions take plase in 2051. The value of multiplicator depends on the minimum geocentric distance in 2051, it may be about 10^4, 10^3.

INTRODUCTION

We investigate possible collisions of asteroids with the Earth, connected with resonant returns. Each possible collision or close approach generate possible approaches or collisions in the future. Asteroid Apophis demonstrate this property. The first list of resonant returns of Apophis was presented in (Chesley, 2006). Possible close approaches and collisions were presented in (Sokolov et al., 2008), (Yeomans et al., 2009), (Chesley, 2011), (Sokolov et al., 2012), (Farnocchia et al., 2013), (Sokolov et al., 2013) as well. Before 2011 returns after approaches in 2029 and 2036 were discussed, later — after 2029 and 2051. Approach in 2051 is located near nominal orbit of Apophis (Chesley, 2011). Resonant returns generate possible collisions not only for Apophis, but for other hazardous asteroids. It is important to search for and investigate all possible collisions.

The list of more than 100 possible collisions of asteroid Apophis in XXI century derived in the Chair of Celestial Mechanics SpbSU is discussed, including relative positions and sizes of gaps, leads to collisions. In spite of orbit refinement in 2013, Apophis will be a hazardous asteroid.

METHOD

To search for the possible collisions of Apophis with the Earth, we applied the Everhart integrator (Everhart, 1974) and Solar System model DE405 (Standish, 1998).

To separate hazardous trajectories of Apophis, we vary the initial conditions in 2006 (JD=2453800.5) and 2035 (JD=2464448.5) . To search for most if not all possible collisions, it is sufficient to change only one variable (mean motion, or semimajor axis, or some coordinate). The method of collisions separation is fully considered in (Sokolov et al. 2008, Sokolov et al. 2012, Sokolov et al. 2013).

For the calculations the high-performance computational cluster of the Saint Petersburg State University used. This cluster consists of 384 computing cores (Intel Xeon X7560). For our task we occupy no more than 64 of all the cores. The program complex utilizes algorithms of parallelization of data flows to optimize amounts of computation time. For the control and the transfer of data the program uses message passing interface technology (realized as Open MPI).

RESULTS

In the Table 1 the ordered positions Δa of Apophis keyholes, dates of collisions, minimum geocentric distances r_{min}, and sizes of keyholes δa are presented. Collisions in XXI century, placed near nominal orbit are considered. First collision take place in 2055; collisions are found each year after 2055 exept 2057, 2063, 2089.

In the Table 2 the ordered positions Δa of important Apophis keyholes, dates of collisions, sizes of keyholes δa, and probabilities P of collisions from NASA website (after probabilities refinement in 2015) are presented.

For the position of any keyhole estimation we use the necessary change Δa (m) of semi-major axis leads to the "main" collision in 2068. Such a relative positions of keyholes one can calculate suffisiently exact and independent on the nominal orbit. Presented in the Table 1 values of Δa and corresponding "relative" values of Sigma LOV, presented in http: / /neo.jpl.nasa.gov /risk /a99942.html, are in good agreement; usually the accuracy of linear approximation is about

0.01. Positions of keyholes forms several groups-clasters: numbers 1-11, 12, 13-16, 17-106, 107-130, 131-140, 141-142, 143-146, 147-148, 149-155, 156-157, 158. Keyholes presented in http: / /neo.jpl.nasa.gov /risk /a99942.html have numbers 21, 92, 95, 98, 108, 124, 135, 145, 156 labelled * , they are labelled triangles in the Figures 1-4. Keyhole labelled "o" correspond to collision in Oktober, not in April.

Regions of initial (JD=2453800.5) semimajor axis without collisions between groups of keyholes have sizes about several tens meters. This sizes change significantly only after close approach to the planet. After 2029 dimensions of regions without collisions, as well as regions leads to collisions, should be multiplied on about 10^5. The next significant change of dimensions take plase in 2051. The value of multiplicator depends on the minimum geocentric distance in 2051. Let consider neighbouring collisions in 2060 (N 21) and in 2055 (N 22); in 2069 (N 123) and in 2068 (N 124).

The first pair: sizes of gaps between 2029 and 2051 is about 10^1 m. Sizes of region between gaps is about 10^5 m. After 2051 this values should be multiplied on about 10^4. Minimum geocentric distances in 2051 are $129 \cdot 10^3$ km (2060) and $93 \cdot 10^3$ km (2055).

The second pair: sizes of gaps between 2029 and 2051 are about 10^0 m (2069) and 10^3 m (2068) Sizes of region between gaps is about 10^4 m. After 2051 this values should be multiplied on about 10^3.. Minimum geocentric distances in 2051 are $783 \cdot 10^3$ km (2069) and $763 \cdot 10^3$ km (2068).

To estimate sizes of the keyholes, the range of semimajor axes δa at 01.05.2035, corresponding to each collision, was calculated. Alternative method of estimation – using the range of minimum geocentric distances Δr_{min} at 2051 for trajectories, corresponding to each collision. In the Table 4 one example is presented. The accuracy of such estimations is about 0.1–0.01.

The collisions with large sizes of keyholes are (number, year): (5, 2075), (11, 2064), (20, 2056), (21*,2060), (22, 2055), (36, 2068), (38, 2068), (40, 2078), (77, 2074), (80, 2098), (85, 2066), (92*, 2065), (95*, 2078), (98*, 2091), (108*, 2077), (112, 2087), (124*, 2068), (135*, 2076), (145*, 2068), (156*, 2069).

Possible collisions characteristics (relative positions and sizes of keyholes) are stable under small change of the model of motion. The stability is considered in (Sokolov et al. 2012). One more example: characteristics of collisions in 2056 and in 2068 ("main" collision, N 124*), derived using "old" (2006) and "new" (2013, after orbit refinement) initial conditions. Collision in 2056. Minimum geocentric distances difference is equal to 24 km. Size of keyhole δa in 2035 difference is equal to 0.4 m. Collision in 2068. Minimum geocentric distances difference is equal to 1.3 km. Size of keyhole δa in 2035 difference is equal to 10 m. Relative positions of keyholes difference is less than 0.1 m. The real accuracy of keyholes characteristics is usually about 10^{-2}.

In the Table 3 relative values of semi-major axes for collisions in 2075, 2056, 2060, 2055, 2069, 2068, 2076 are presented at 06.03.2006, 28.01.2015, 01.05.2035, 23.04.2052. Relative semi-major axis is the difference between the semi-major axis and the semi-major axis, corresponding to the "main" collision in 2068 at the same time.

CONCLUSIONS

The positions of keyholes demonstrate complicated structure similar to fractals structure due to resonant returns of asteroids. This complicated structure must be taken into account in the design of collisions eliminating.

There are many keyholes near Apophis nominal orbit. In spite of orbit refinement in 2012-2013, Apophis will continue a hazardous asteroid.

Our results are in agreement with the results, presented in (Yeomans et al., 2009), (Chesley, 2011), (Farnocchia et al., 2013).

ACKNOWLEDGMENT

This work was partly supported by the Russian Foundation of Basic Research, project no. 14-02-00804, project no 15-02-04340, and by Saint-Petersburg State University grant N 6.37.341.2015.

The author thank N.A. Petrov and A.A. Vasiliev for helpful cooperation.

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Table 1: Possible collisions of asteroid Apophis with the Earth

N	Δa(m)	Date	r_{min}(km)	δa(m)
1	-198.885	2097 4 12.6538	22.43	0.167D+01
2	-195.379	2098 4 13.0754	3954.37	0.563D+00
3	-194.337	2073 4 13.1850	4772.19	0.130D+00

Table 1: Possible collisions of asteroid Apophis with the Earth

N	Δa(m)	Date	r_{min}(km)	δa(m)
4	-194.307	2083 4 13.3130	1029.76	0.187D+01
5	-193.713	2075 4 13.2101	49.24	0.282D+02
6	-193.712	2085 4 12.8557	6138.12	0.380D-01
7	-193.690	2084 4 12.6336	211.76	0.896D+00
8	-192.621	2072 4 12.8571	3800.03	0.915D+00
9	-192.606	2073 4 13.1118	1325.43	0.111D+01
10	-192.389	2059 4 13.6741	3478.36	0.366D+01
11	-190.754	2064 4 13.0235	4575.66	0.129D+02
12	-119.820	2084 4 11.9460	3354.50	0.113D+00
13	-87.675	2064 4 12.9994	5534.07	0.604D+00
14	-87.638	2074 4 12.9038	38.47	0.708D+00
15	-87.586	2058 4 13.3884	4425.72	0.124D+00
16	-87.537	2062 4 13.4094	3033.00	0.674D+00
17	-76.703	2067 4 13.7233	5811.75	0.206D+00
18	-71.103	2060 4 12.8202	1414.28	0.995D-01
19	-71.095	2065 4 13.1981	5053.37	0.942D-01
20	-71.095	2056 4 13.0972	4204.05	0.798D+02
21*	-67.384	2060 4 13.0882	4404.01	0.205D+02
22	-64.712	2055 4 13.8186	5921.09	0.121D+02
23	-60.129	2084 4 12.0410	814.89	0.103D+01
24	-59.881	2095 4 13.9578	696.74	0.417D+00
25	-59.797	2080 4 13.4384	1565.63	0.294D+01
26	-59.781	2092 4 13.3853	1545.92	0.119D+00
27	-59.691	2097 4 13.6979	2428.35	0.600D-01
28	-59.556	2092 4 13.3852	1546.21	0.102D+00
29	-59.491	2088 4 13.1162	293.71	0.890D-01
30	-59.486	2074 4 13.9537	2935.79	0.206D+01
31	-59.429	2084 4 13.4086	1435.08	0.320D+01
32	-59.045	2078 4 13.7278	1148.50	0.305D+01
33	-58.985	2073 4 13.4591	1399.42	0.414D+01
34	-58.858	2090 4 13.7400	2338.27	0.551D+00
35	-58.850	2080 4 13.0950	1821.87	0.181D+00
36	-58.847	2068 4 13.2084	556.93	0.807D+01
37	-58.625	2095 4 13.7602	1400.45	0.480D-01
38	-58.511	2068 4 13.1972	34.34	0.817D+01
39	-58.406	2073 4 13.4308	284.43	0.296D+01
40	-58.360	2078 4 13.6827	19.86	0.183D+02
41	-58.351	2083 4 13.9059	31.89	0.180D+01
42	-57.980	2078 4 13.6655	2060.35	0.540D+00
43	-57.961	2070 4 13.6113	5315.45	0.379D+00
44	-57.924	2062 4 13.6250	39.14	0.123D+01

Table 1: Possible collisions of asteroid Apophis with the Earth

N	Δa(m)	Date	r_{min}(km)	δa(m)
45	-57.891	2065 4 13.3622	5353.07	0.380D+00
46	-57.825	2077 4 13.3748	1088.82	0.420D+00
47	-57.700	2073 4 13.3901	1516.30	0.341D+00
48	-57.695	2070 4 13.6098	5590.58	0.167D+00
49	-57.678	2067 4 13.8508	5509.12	0.238D+00
50	-57.676	2094 4 13.5618	1737.94	0.268D+00
51	-57.623	2061 4 13.3595	2599.14	0.736D+00
52	-57.617	2088 4 13.0213	5765.85	0.270D-01
53	-57.589	2058 4 13.5975	5517.28	0.429D+00
54	-57.499	2082 4 13.5806	5078.33	0.145D+00
55	-57.356	2075 4 13.8662	3130.60	0.250D+00
56	-57.319	2072 4 13.1252	2375.99	0.273D+00
57	-57.308	2067 4 13.8467	1590.42	0.344D+00
58	-57.306	2078 4 13.6044	2033.31	0.235D+00
59	-57.297	2062 4 13.5896	5263.99	0.248D+00
60	-57.212	2071 4 13.8438	307.33	0.311D+00
61	-57.046	2099 4 13.6277	2730.78	0.810D-01
62	-57.021	2066 4 13.5893	3650.09	0.311D+00
63	-57.013	2077 4 13.2630	3971.24	0.268D+00
64	-57.001	2098 4 13.4266	2366.52	0.890D-01
65	-56.997	2067 4 13.8445	2020.71	0.324D+00
66	-56.847	2092 4 12.9144	2970.11	0.119D+00
67	-56.795	2078 4 13.5138	46.09	0.339D+00
68	-56.570	2068 4 13.0709	1245.96	0.753D+00
69	-56.509	2077 4 13.1509	23.40	0.890D-01
70	-55.995	2095 4 13.1282	2149.08	0.607D+00
71	-55.830	2098 4 13.0267	5191.02	0.300D+01
72	-54.764	2096 4 12.1486	3003.12	0.240D-01
73	-54.399	2092 4 17.9170	5099.20	0.570D-01
74	-54.286	2088 4 12.7328	595.74	0.360D+01
75	-47.140	2079 10 16.8086	1214.97	0.586D+01
76	-47.094	2099 4 13.4968	5813.57	0.239D+00
77	-46.339	2074 4 13.3568	30.01	0.192D+02
78	-46.173	2095 4 13.4856	2823.12	0.447D+00
79	-46.148	2099 4 13.3636	1895.03	0.330D-01
80	-46.108	2098 10 16.4792	1232.91	0.439D+02
81	-45.530	2090 10 16.5753	225.96	0.663D+01
82	-43.872	2090 4 13.2377	1350.79	0.570D-01
83	-43.568	2081 4 12.9635	898.15	0.324D+00
84	-43.077	2097 4 13.0624	5657.34	0.600D-01
85	-42.739	2066 4 13.5824	5239.31	0.186D+02

Table 1: Possible collisions of asteroid Apophis with the Earth

N	Δa(m)	Date	r_{min}(km)	δa(m)
86	-42.700	2093 4 12.9521	34.93	0.533D+00
87	-42.699	2080 4 12.7770	579.22	0.144D+01
88	-41.638	2080 4 12.7689	4931.10	0.165D+01
89	-41.638	2096 4 12.7097	1732.76	0.154D+00
90	-41.610	2081 4 13.0546	16.65	0.712D+00
91	-41.599	2082 4 13.3903	2779.98	0.227D+00
92*	-41.586	2065 4 13.1313	2866.35	0.392D+02
93	-40.848	2093 4 13.0271	3851.17	0.696D+00
94	-40.817	2094 4 13.3006	4060.79	0.200D+00
95*	-40.803	2078 4 13.3755	371.03	0.409D+02
96	-40.797	2079 4 13.3975	29.71	0.121D+00
97	-40.070	2095 4 13.0606	4857.94	0.120D-01
98*	-40.003	2091 4 13.3708	5454.01	0.928D+01
99	-39.765	2075 4 13.4812	735.39	0.751D+00
100	-36.182	2099 4 13.4016	2198.58	0.147D+00
101	-36.173	2098 4 12.9238	2342.26	0.415D+00
102	-29.538	2099 4 13.1452	654.05	0.164D+01
103	-29.517	2075 4 13.7053	6084.62	0.918D-01
104	-24.816	2097 4 13.0938	6108.24	0.360D-01
105	-16.559	2086 4 13.1801	5027.57	0.769D+00
106	-11.722	2088 10 15.7617	29.48	0.788D+00
107	-1.273	2085 4 13.0042	3504.85	0.261D+00
108*	-1.266	2077 4 13.1125	2484.03	0.382D+02
109	-1.211	2096 4 12.7238	2262.32	0.956D+00
110	-0.727	2094 4 12.9472	3899.60	0.208D-01
111	-0.724	2096 4 12.5436	161.86	0.346D+00
112	-0.713	2087 4 13.3967	407.74	0.822D+01
113	-0.516	2097 4 12.8211	3716.23	0.163D+00
114	-0.501	2078 4 13.2656	5816.31	0.241D+01
115	-0.370	2081 4 12.4202	3023.70	0.600D-02
116	-0.318	2071 4 12.9930	3631.28	0.600D-01
117	-0.194	2095 4 13.0993	2048.82	0.773D+00
118	-0.164	2082 4 12.9246	531.68	0.829D+00
119	-0.164	2083 4 13.1321	1299.23	0.713D+00
120	-0.146	2084 4 12.4360	243.11	0.600D+00
121	-0.116	2092 4 12.3518	21.32	0.198D+00
122	-0.097	2093 10 16.0022	3146.41	0.292D+00
123	-0.063	2069 4 12.7248	3107.42	0.135D+01
124*	0.000	2068 4 12.6318	40.28	0.153D+04
125	0.008	2072 4 12.2427	2326.97	0.555D-01
126	0.085	2073 4 20.6818	43.23	0.413D+00

Table 1: Possible collisions of asteroid Apophis with the Earth

N	Δa(m)	Date	r_{min}(km)	δa(m)
127	0.134	2074 4 12.9464	1272.28	0.681D+00
128	0.233	2075 4 13.1979	880.33	0.204D+01
129	0.273	2090 4 12.9052	5004.67	0.755D+00
130	0.494	2093 4 12.2460	282.57	0.663D+00
131	33.725	2084 4 12.4547	873.08	0.202D+01
132	33.775	2098 4 12.9696	2269.06	0.261D+00
133	33.797	2083 4 13.2189	1339.86	0.370D+00
134	33.803	2082 4 13.0056	42.28	0.141D+00
135*	33.818	2076 4 12.6966	702.03	0.150D+03
136	33.873	2095 5 7.3834	57.56	0.755D+00
137	33.914	2085 4 13.1116	3431.90	0.272D+01
138	35.685	2073 4 13.0516	3480.03	0.700D+00
139	35.846	2060 4 12.9937	4862.49	0.144D+01
140	38.370	2099 4 13.3483	6294.30	0.188D+00
141	121.920	2095 4 13.3202	6056.35	0.597D-01
142	124.609	2092 4 12.5181	4704.72	0.588D-01
143	1606.715	2099 4 14.5881	2154.24	0.298D+00
144	1607.640	2096 4 14.8829	24.81	0.127D+00
145*	1610.591	2068 10 15.3232	2973.61	0.686D+02
146	1614.010	2092 4 10.8109	2376.54	0.192D+01
147	1684.927	2096 4 12.1659	3125.46	0.292D-01
148	1686.110	2087 4 13.0017	1763.37	0.178D+01
149	3465.530	2094 4 6.1114	212.59	0.212D+00
150	3465.709	2098 4 4.8156	35.19	0.140D+01
151	3465.711	2099 4 5.0600	661.94	0.892D+00
152	3465.714	2082 4 4.7226	192.24	0.157D+01
153	3472.360	2094 4 9.1062	2660.40	0.456D+00
154	3472.367	2093 4 17.6726	2679.21	0.800D+00
155	3472.919	2087 4 9.1628	1058.61	0.468D+00
156*	4697.565	2069 10 15.5930	374.42	0.267D+03
157	4712.480	2090 10 15.6852	5710.46	0.169D+00
158	4777.404	2097 4 11.9934	1229.65	0.522D+01

Table 2: Important gaps of Apophis

N	Date	$\Delta a(m)$	$\delta a(m)$	$P \cdot 10^7$(NASA)
1*	2089o	-3592.1	23.	0.0010
2	2075	-193.7	28.	
3	2064	-190.8	13.	
4	2056	-71.1	80.	
5*	2060	-67.4	21.	1.0
6	2055	-64.7	12.	
7	2078	-58.4	18.	
8	2074	-46.3	19.	
9	2098	-46.1	44.	
10	2066	-42.7	19.	
11*	2065	-41.6	39.	2.6
12*	2078	-40.8	41.	2.2
13*	2091	-40.0	9.3	2.2
14*	2077	-1.27	38.	1.8
15	2087	-0.71	8.2	
16	2069	-0.06	1.4	
17*	2068	0.00	1530.	67.
18*	2076	33.8	150.	5.4
19*	2068o	1611.	69.	0.000079

Table 3: Time dependence of relative semi-major axes of collisions (m)

Date	06.03.06	28.01.15	01.05.35	23.04.52
2075	-194.	-137.	$4.38 \cdot 10^7$	$3.15 \cdot 10^9$
2056	-71.3	-50.3	$1.61 \cdot 10^7$	$11.3 \cdot 10^9$
2060	-67.5	-47.7	$1.52 \cdot 10^7$	$14.7 \cdot 10^9$
2055	-64.8	-45.8	$1.46 \cdot 10^7$	$19.2 \cdot 10^9$
2069	-0.075	-0.052	$14.8 \cdot 10^3$	$-0.254 \cdot 10^7$
2068	0.0	0.0	$0.0 \cdot 10^0$	$0.0 \cdot 10^0$
2076	33.9	24.0	$-0.764 \cdot 10^7$	$0.833 \cdot 10^9$

Table 4: Estimations of keyholes sizes

Date	$\delta a(35)$(m)	$\Delta r_{min}(51)$(km)	$100\delta a/\Delta r_{min}$
2055	12.1	1.76	0.69
2060	20.5	2.66	0.77
2068	1530	208	0.74
2069	1.53	0.19	0.71

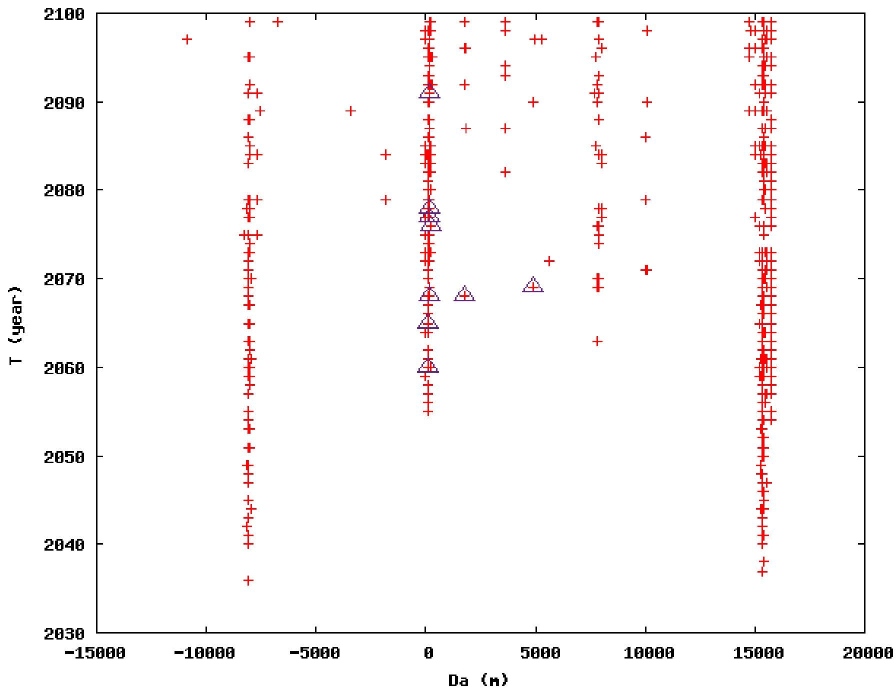

Figure 1: The relative position Da of the gaps leading to collisions of Apophis with the Earth and dates of collisions.

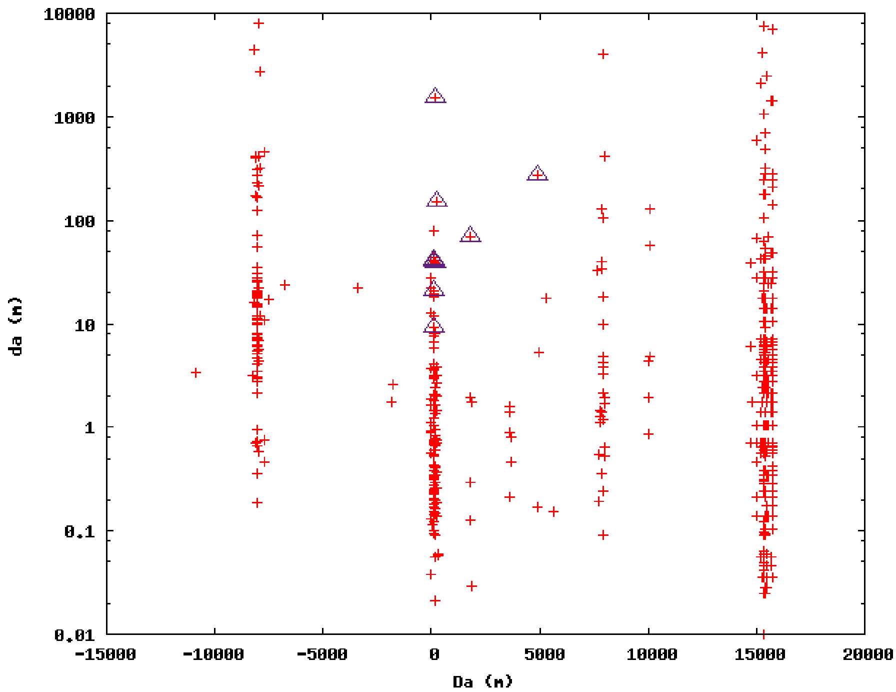

Figure 2: The relative position Da and size da of the gaps leading to collisions of Apophis with the Earth.

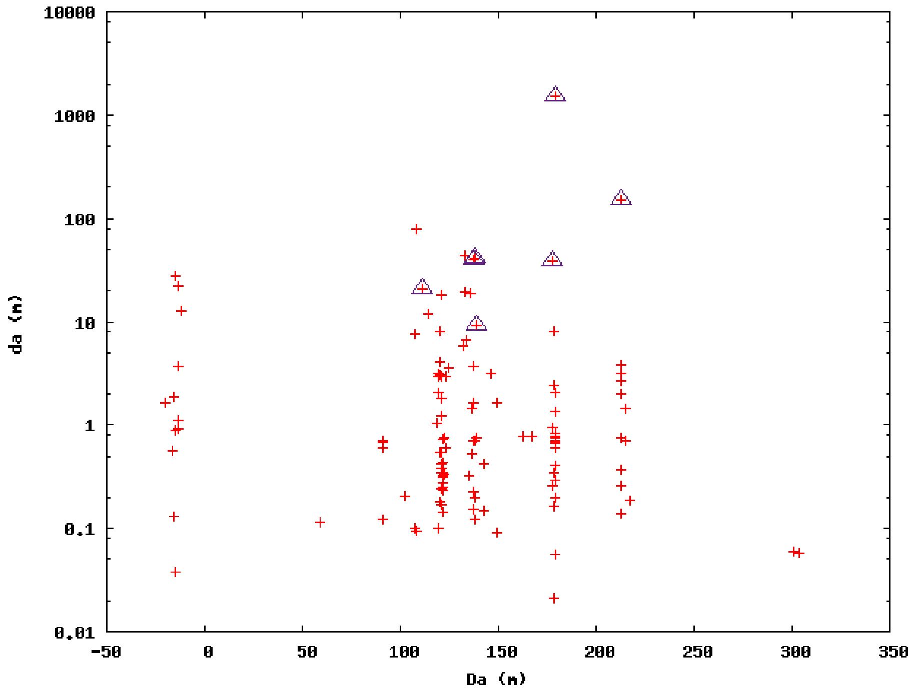

Figure 3: The relative position Da and size da of the gaps leading to collisions of Apophis with the Earth in the vicinity of the nominal orbit.

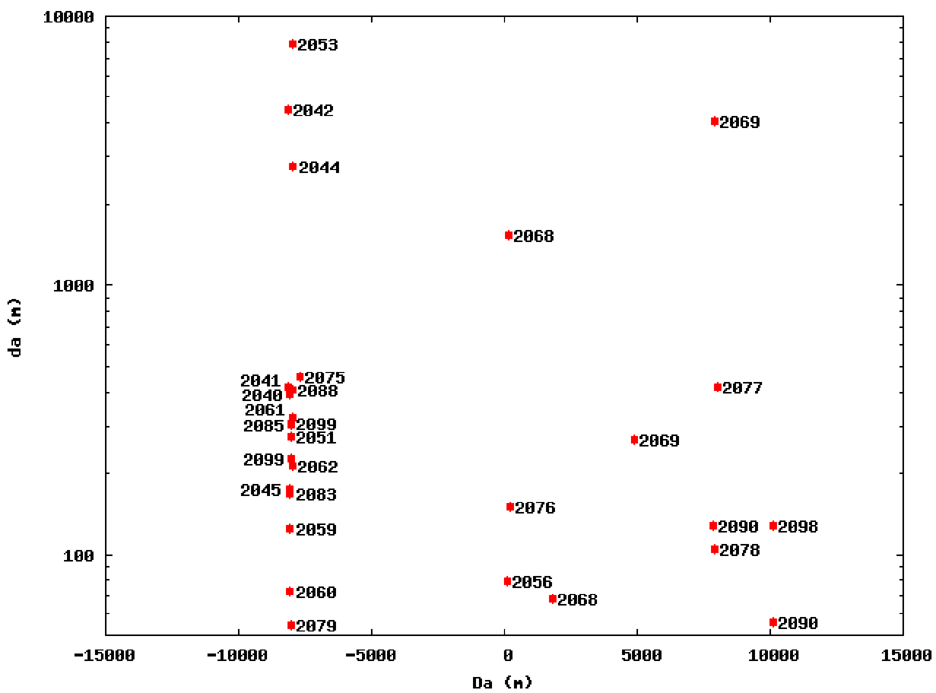

Figure 4: The relative position Da and size da of the large gaps leading to collisions of Apophis with the Earth and dates of collisions.