Efficient and Accurate Mapping of Nuclear-Energy Deposition

Robert Managan, Kirsten Howley, & Joseph Wasem
Lawrence Livermore National Laboratory, Livermore, CA, USA

Abstract

Calculating the energy deposition from a nuclear explosion in an asteroid is a time-intensive process if a very accurate solution is desired. The problem becomes even more cumbersome if a parametric study involving many orientations and heights of burst (HOB) is considered. Because the explosion takes place in a vacuum the deposition is determined by the angle of incidence and the spectrum of the radiation at any given point on the surface. This means that the energy deposition is a one-dimensional problem at each point on the surface. These 1D problems can provide an accuracy unaffordable in 3D calculations. Moreover 1D solutions enable resolving the short absorption length which is a characteristic of keV energy x-rays.

We will present the approach of taking 1D deposition solutions and mapping them onto 2 or 3 dimensional shapes. For some sources the 1D calculations at each angle of incidence, χ, can be approximated by an analytic profile. For others the tabular deposition versus depth and angle of incidence can be interpolated to find the energy deposition at any point inside the target based on the depth to the nearest surface point. Both methods are fast and can be applied repeatedly to the different scenarios of a parametric study.

Sources and Targets

To model the energy deposition from a nuclear explosive we consider x-rays and neutrons of various energies. Monochromatic sources are easier to analyze than black body or general spectra. Nevertheless we consider here black body spectra with temperatures of 1.0 and 2.0 keV and monochromatic neutron sources with energies of 2.45 MeV and 14.1 MeV. A real device would have a spectrum of neutron energies, however the energies considered here illustrate the relevant physics of deposition.

We considered four different materials to cover the variability of asteroid and comet compositions: (such as) ice, quartz (SiO_2), forsterite (Mg_2SiO_4), and iron-nickel to name a few. For this study results for SiO_2 are presented.
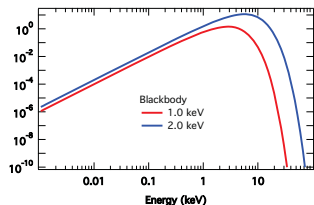
Mapping 1D to 2D etc.

This figure shows the relevant angles and lengths used in the mapping. The angle of incidence is χ.
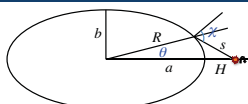
When the HOB and the size of the asteroid (R_0) change two scaling rules will ensure the energy deposition scales the same way.
1. The yield of the source must increase by $(R/R_0)^2$ so the fluence at the surface of the asteroid remains the same.
2. The density must decrease by (R_0/R) so the absorption length increases by (R/R_0) to match the increase of the rest of the geometry.

The energy density inside the asteroid depends on the depth below the surface, the angle of incidence at the surface, and ϵ, the fluence at the surface. Mercury[1] and MCNP[2] are used to calculate the energy deposition, normalized by the incident energy, in a 1D column for a range of incidence angles, call it $F(r, \chi)$. Then the energy density in a 2D or 3D problem is calculated using

$$\epsilon(\theta) = \frac{Y}{4\pi s^2} \cos\chi$$
$$E(r, \theta) = F(r, \chi)\epsilon(\theta)$$

For 1 and 2 keV black body x-rays the energy deposition profiles (shown below) can be modeled as a combination of several exponential functions. This analytic form works since the reflected energy does not vary with χ (not including any re-radiation).

$$f(x, \chi) = \frac{1}{\cos\chi}\left(a_1 e^{-\frac{x}{a_2\cos\chi}} + a_3 e^{-\frac{x}{a_4\cos\chi}} + a_5 e^{-\frac{x}{a_6\cos\chi}} + a_7 e^{-\frac{x}{a_8\cos\chi}}\right)$$
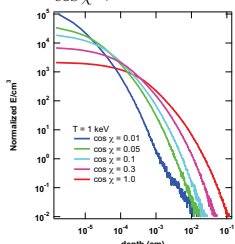
| $a_1 = 1639.7$	$a_2 = 1.7174e-4$
$a_3 = 427.575$	$a_4 = 1.1281e-3$
$a_5 = 37.387$	$a_6 = 5.539e-3$
$a_7 = 1.3624$	$a_8 = 2.126e-2$
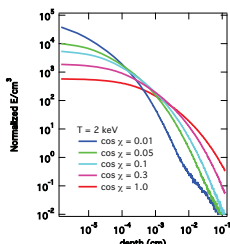
$a_1 = 344.639$	$a_2 = 1.3295e-4$
$a_3 = 183.078$	$a_4 = 7.3757e-4$
$a_5 = 85.873$	$a_6 = 3.5958e-3$
$a_7 = 22.746$	$a_8 = 1.3934e-2$

Mapping (cont.)

For the neutron sources the amount reflected can be significant and varies with the incident angle. In addition there are exo- and endo-thermic neutron reactions that increase the deposited energy above or below that of the source energy. The gammas that result from the capture reactions on Si deposit their energy about 100 cm deep in SiO_2. The 14.1 MeV neutrons are above the threshold for spallation reactions which reduce the deposited energy.
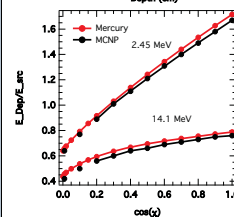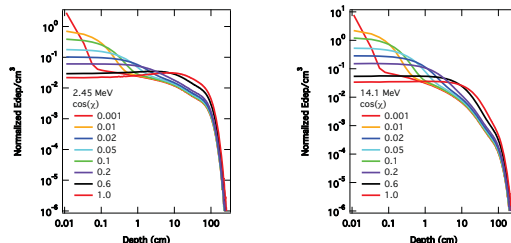

Here the 2.45 MeV 1D deposition profiles are mapped onto an asteroid with a radius of 10 m from a HOB of 4 m. The small radius was used so that the deposition depth is visible along with the curvature of the surface.
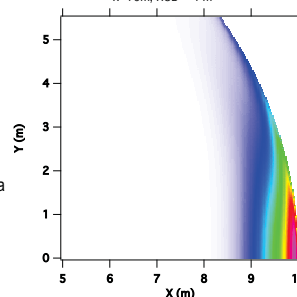
Applications

One application of this method is to calculate the height of burst (HOB) that maximizes the volume of melted material given the melt threshold, 1941 J/g for SiO_2. The melted material is a conservative estimate of the mass involved in the blow-off and maximizing this mass is an estimate of the HOB that maximizes the imparted momentum[3]. Below are the results as a function of yield and shape of the asteroid. The minimum in the optimal HOB occurs for oblate spheroids because you gain solid angle subtended more quickly than for spheres. The contours are labeled in meters.
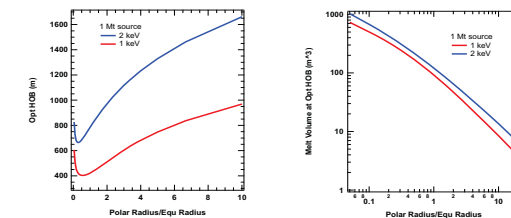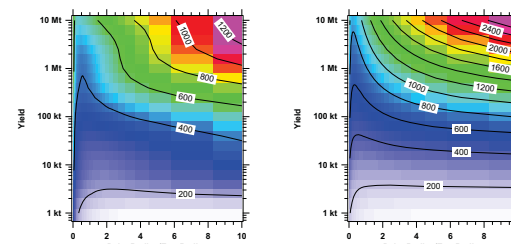
Conclusions

- Mapping energy deposition from 1D solutions to 2D or 3D shapes is a well-posed problem and is efficient.
- In 1D one can explore the micron scale deposition profile generated by low energy x-rays and analyze the gain or loss of source energy due to nuclear reactions.
- Scaling rules for self-similar deposition profiles are given.
- After calculating deposition profiles once for a given material you can rapidly explore many variations of yield, height of burst, shape, and orientation quickly and efficiently.

[1] P. S. Brantley, S. A. Dawson, M. S. McKinley, M. J. O'Brien, R. J. Procassini, S. M. Sepke and D. E. Stevens, *Mercury User Guide:Version d.8*, LLNL-SM-560687, Lawrence Livermore National Laboratory (2012).
[2] Los Alamos National Laboratory, http://www.mcnpx.lanl.gov.
[3] J. M. Owen, A Comparison of Kinetic Impactor And Nuclear Deflection For Two Scenarios: Bennu And The 2015 PDC Scenario, IAA-PDC-15-03-08.