Internal Gravity, Self-Energy, and Disruption of Asteroids

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As a part of the NASA Ames effort to characterize Near-Earth Asteroids for NASA's new Planetary Defense project, we are modeling the energy required to disrupt potentially hazardous asteroids and comets with realistic shapes. This physics-based model will be an input to the other tasks in the ARC project, as well as to outside projects such as asteroid hazard mitigation strategies.

DISRUPTION

The *specific energy of disruption* Q_D^* is defined as the minimum energy E_D required to shatter a body and to remove at least half of its mass, divided by the total mass M of the original object. This specific disruption energy may be regarded as the sum of three terms: $Q_D^* = Q_S^* + Q_G^* - Q_R^*$, where D stands for disruption, S for shattering, G for gravitation, and R for rotation.

SHATTERING

Here Q_S^* is the energy input per unit mass required just to shatter the body. Q_S^* depends on the strength, density, and porosity of the material composing the body, and may also decrease with its size due to the "weakest link" effect, as shown in FIGURE 1. Bodies smaller than ~ 5 km in radius are likely to be "strength-dominated", while larger ones are likely to be "gravity-dominated".

Fig. 1. A body's specific disruption energy Q_D^* as a function of its mean radius. The solid line represents Q_S^*, the body's specific disruption energy due to its internal strength. The dashed line represents the body's specific gravitational binding energy Q_G^*, while the dotted curve represents their sum $Q_S^* + Q_G^*$.

GRAVITATIONAL BINDING ENERGY

$Q*_G \equiv E_G/M$ is the specific self-gravitational binding energy of a body. This binding energy E_G may be obtained by integrating the object's internal gravitational potential Φ over its entire mass. Fig. 1 also shows that Q_G^* scales as the square of the object's mean radius:

$$Q_G = FG\bar{\rho}\bar{R}^2, \tag{1}$$

where G is Newton's constant of universal gravitation, $\bar{\rho}$ is the body's mean density, and $\bar{R}$ is its mean radius, defined as the radius of a sphere with the same volume. The coefficient F is a dimensionless "form factor" depending only on the shape of the object.

Analytic or semi-analytic formulae are not known for most shapes, but any solid body can be approximated to arbitrary accuracy by a polyhedron of sufficient complexity. An analytic formula is known for the gravitational potential Φ of any homogeneous polyhedron. This enables us to find semi-analytically the self-energy E_G, specific binding energy Q_G, and gravitational form factor F of any polyhedron.

POLY-HEDRON	TETRA-HEDRON	CUBE	OCTA-HEDRON	DODECA-HEDRON	ICOSA-HEDRON	SPHERE
FACES	4	6	8	12	20	∞
EDGES	6	12	12	30	30	∞
VERTICES	4	8	6	20	12	∞
FORM FACTOR	2.3017	2.4453	2.4575	2.4982	2.5027	2.51327

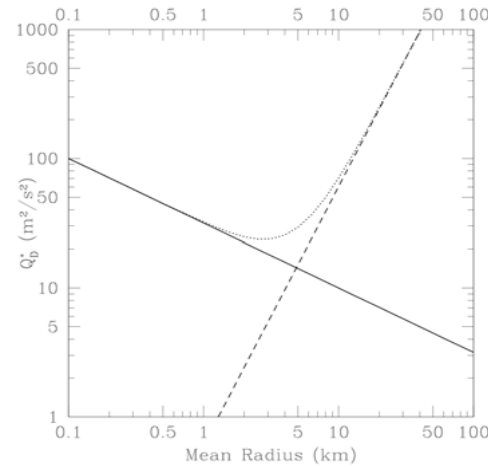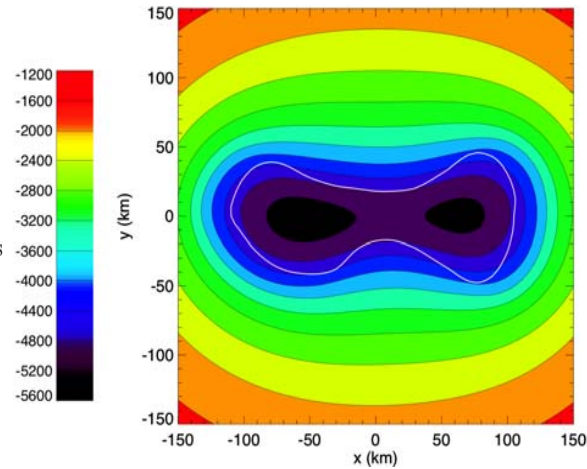Table 1. Gravitational form factors F of homogeneous Platonic solids and spheres.

Fig. 2. Gravitational potential
of ham-bone-shaped asteroid 216
Kleopatra in its equatorial plane,
assuming that its density is uniform.
Note how the potential is continuous
across its surface,
denoted by the white outline.
The scale bar is labeled in m²/s².

Knowledge of its internal gravity also assists analysis of its interior stresses and strains.

For example, FIGURE 2 contours the gravitational potential Φ in the equatorial plane of the ham-bone-shaped asteroid 216 Kleopatra, assuming that its density ρ is uniform. Integrating this Φ over Kleopatra's volume enables us to find E_G, Q_G^*, and $F \approx 2.05$. For comparison, TABLE 1 lists the gravitational form factors for a homogeneous sphere and all five Platonic solids. Note how high all of these form factors are compared to Kleopatra's.

As a further comparison, FIGURE 3 contours the gravitational form factor F as a function of the aspect ratios b/a and c/b for ellipsoids with principal radii $a \geq b \geq c$, while FIGURE 4 does the same for cuboids with sides $a \geq b \geq c$. In either case, form factors of 2.05 like Kleopatra's require extreme aspect ratios on the order of 1/4, roughly consistent with Kleopatra's shape. This confirms that Kleopatra is weakly bound gravitationally because of its extreme elongation.

ENERGY REBATE

Finally, in calculating the gravitational binding energy E_G, we have assumed that the body's entire mass M is dispersed to infinity. In fact, it suffices to re-arrange the body into two widely separated spheres, each with half of the mass M of the original object. This saves energy from E_G and reduces the gravitational form factor F by a considerable amount $2\pi \sqrt[3]{2}/5 \approx 1.583$. For example, the form factor of a non-rotating sphere would be reduced from 2.513 to 0.930, while that for Kleopatra would be reduced from ~ 2.00 to only ~ 0.620 . By means of this "energy rebate" and/or rotational energy, it is possible to reduce the gravitational form factor to zero.
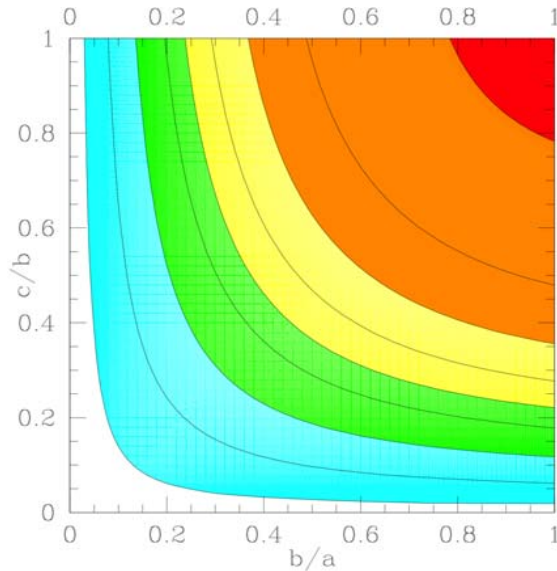
ROTATION

The third contribution to a body's specific disruption energy Q_D^* is its specific energy of rotation Q_{*R}. Note that the contribution of this term to the form factor F also scales as the square of the body's mean radius, but that its contributions to Q_D^* and F both are *negative*. For example, Kleopatra rotates with a period of 5.385 hours; this corresponds to a contribution of only -0.049 to the form factor F, leaving a total of $F \approx 2.00$.
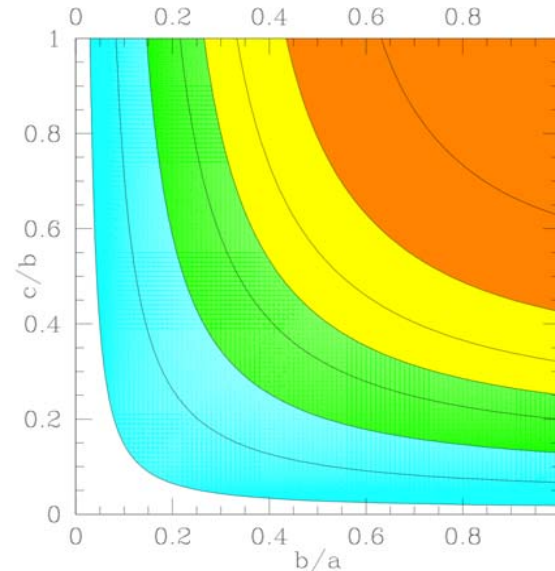

Fig. 3. Gravitational form factor F

for ellipsoids of principal radii $a \geq b \geq c$.

Fig. 4. Gravitational form factor F for cuboids

(rectangular parallelepipeds) of sides $a \geq b \geq c$.