INTERNAL GRAVITY, SELF-ENERGY, AND DISRUPTION OF ASTEROIDS Anthony R. Dobrovolskis[1], Donald G. Korycansky[2], [1]SETI Institute, 245-3 NASA Ames Research Center, Moffett Field, CA 94035-1000, USA; ; [2] CODEP, Dept. of Earth Sciences, U. of California, Santa Cruz, CA 95064-1077, USA

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As a part of the NASA Ames effort to characterize Near-Earth Asteroids for NASA's new Planetary Defense project, we are modeling the energy required to disrupt potentially hazardous asteroids and comets with realistic shapes. This physics-based model will be an input to the other tasks in the ARC project, as well as to outside projects such as asteroid hazard mitigation strategies.

DISRUPTION

A basic concept for asteroid collisional evolution as well as some mitigation strategies is Q^*_D, the *specific energy of disruption* of a comet or asteroid. This is defined as the minimum energy input E_D required to shatter the body and to remove at least half of its mass, divided by the total mass M of the original object. This specific disruption energy may be regarded as the sum of three terms:

$$Q^*_D = Q^*_S + Q^*_G - Q^*_R, \qquad (1)$$

where D stands for disruption, S for shattering, G for gravitation, and R for rotation.

SHATTERING

Here Q^*_S is the energy input per unit mass required just to shatter the body into many small pieces. In this context, the meaning of "many" and "small" depends on circumstances. For example, many comets and asteroids (such as Halley and Itokawa) are "bilobate" - that is, they resemble two parts stuck together in a kind of snowman shape. To shatter a "contact binary" composed of two identical spheres, it suffices to break the two lobes apart at their connection point. Other objects may already be shattered into thousands of pieces by non-disruptive impacts, or may be "rubble piles" of boulders and gravel with no significant cohesion.

However, most asteroids smaller than $\sim$ 75 meters in radius may be solid, monolithic bodies. For such objects, Q^*_S depends on the strength, density, and porosity of the material composing the body. Simple models suggest that Q^*_S should be independent of the size of the object, but more sophisticated reasoning reveals that Q^*_S should decrease with increasing size, based on the "weakest link" effect. Several arguments suggest that Q^*_S should scale roughly as the inverse square root of the body's mean radius. FIGURE 1 (in a similar format to Fig. 1 of Asphaug *et al.* 2002) compares the above strength scaling with the naïve constant-Q^*_S model.

As Fig. 1 shows, comets and asteroids smaller than $\sim$ 5 km in radius are likely to be "strength-dominated";

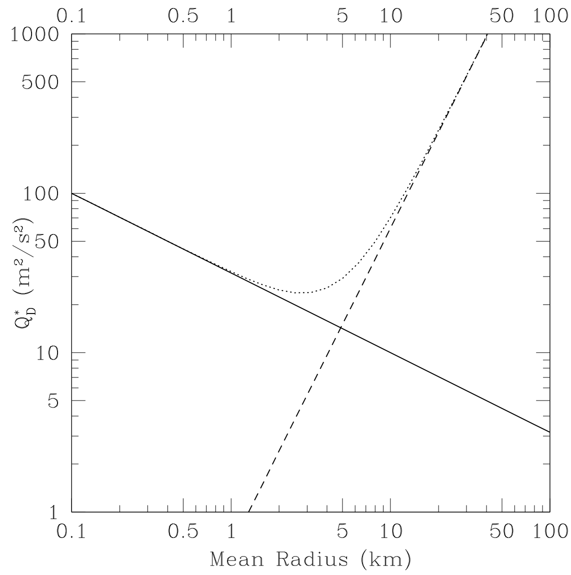

Figure 1: A body's specific disruption energy Q^*_D as a function of its mean radius. The solid line represents Q^*_S, the body's specific disruption energy due to its internal strength. The dashed line represents the body's specific gravitational binding energy Q^*_G, while the dotted curve represents their sum $Q^*_S + Q^*_G$.

that is, their specific shattering energy Q^*_S exceeds their specific gravitational binding energy Q^*_G. The reverse applies to those larger than $\sim$ 5 km in radius, which are thus likely to be "gravity-dominated". Here Q^*_G is defined as E_G/M, where E_G is the body's self-gravitational energy; that is, the energy input it would take to disperse the entire mass to infinity.

GRAVITATIONAL BINDING ENERGY

This binding energy E_G may be obtained by integrating the object's internal gravitational potential Φ over its entire mass:

$$E_G = -\frac{1}{2}\int \Phi(\mathbf{r})dM(\mathbf{r}), \qquad (2)$$

where the mass element $dM(\mathbf{r})$ is just $\rho(\mathbf{r})dxdydz$, and $\rho(\mathbf{r})$ is the body's mass density at interior point $\mathbf{r}$. The leading factor of 1/2 in formula (2) above compensates for the fact that the integration counts each pair of interior points twice. Note that by this sign convention, gravitational potential Φ is negative, so the self-gravitational energy E_G and specific energy $Q^*_G \equiv E_G/M$ both are positive.

Fig. 1 also shows that Q^*_G scales as the square of the object's mean radius:

$$Q^*_G = FG\bar{\rho}\bar{R}^2, \quad (3)$$

where G is Newton's constant of universal gravitation, $\bar{\rho}$ is the body's mean density (its total mass M divided by its volume V), and $\bar{R}$ is its mean radius, defined as $\sqrt[3]{\frac{3V}{4\pi}}$, the radius of a sphere with the same volume V. The coefficient F may be considered as a dimensionless "form factor" depending only on the shape of the object.

Analytic or semi-analytic formulae are known for the self-gravitational energy E_G of a homogeneous ellipsoid (Neutsch 1979), a homogeneous cuboid (rectangular parallelepiped; Waldvogel 1976), and a "duplex" of two homogeneous spheres stuck together (Dobrovolskis and Korycansky, in preparation). Such formulae are not yet known for the self-energy E_G of other figures, but any solid body such as a comet or asteroid can be approximated to arbitrary accuracy by a polyhedron of sufficient complexity. An analytic formula is known for the gravitational potential Φ of any homogeneous polyhedron (*e.g.* Waldvogel 1979, Werner 1994), but it is widely believed that the formula for Φ applies only on the surface or outside of the object (*e.g.* Werner 1994, Werner and Scheeres 1997).

We show instead that this formula applies equally well inside the body. This enables us to find the internal potential Φ, self-energy E_G, specific binding energy Q^*_G, and gravitational form factor F of any polyhedron. Knowledge of its internal gravity also assists analysis of its interior stresses and strains (*e.g.* Dobrovolskis 1982). For example, FIGURE 2 contours the gravitational potential Φ in the equatorial plane of M-type asteroid 216 Kleopatra, assuming that its density ρ is uniform. Note that Kleopatra is notoriously shaped like a ham-bone, as shown by the heavy contour representing its surface (Ostro *et al.* 2000, Descamps *et al.* 2011).

Note also that the gravitational potential Φ remains continuous as it crosses Kleopatra's surface from exterior to interior. Integrating this Φ over Kleopatra's volume, and multiplying by its mean density $\bar{\rho} \approx 3.6$ kg/ℓ, gives its gravitational self-energy $E_G \approx 1.04 \times 10^{22}$ Joules. Then dividing again by Kleopatra's mass $M \approx 4.64 \times 10^{18}$ kg gives $Q^*_G \approx 2250$ m^2/s^2, while formula (2) above and Kleopatra's mean radius $\bar{R} \approx 67.5$ km finally give its gravitational form factor $F \approx 2.05$.

For comparison, Table 1 lists the form factors for a homogeneous sphere and all five Platonic solids. The sphere has the highest form factor $F = 4\pi/5 \approx 2.513$, because the sphere is the figure with the greatest binding energy for a given mass and volume; but all of the other shapes have comparable values, because they all are equidimensional. Even a contact binary composed of two identical spheres has $F = \frac{17\pi}{15\sqrt[3]{4}} \approx 2.243$. Note how high all of these form factors are compared to Kleopatra's.

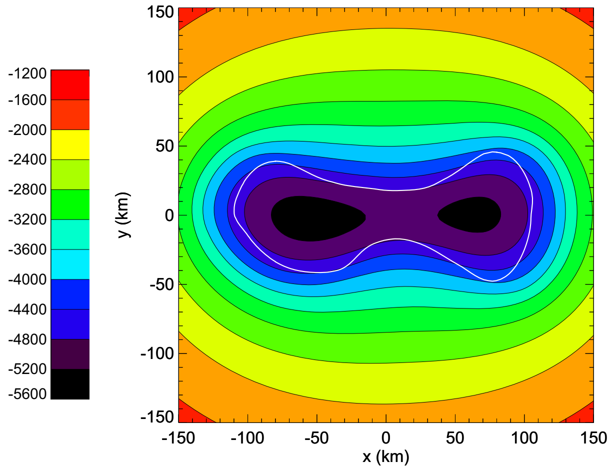

Figure 2: Gravitational potential of ham-bone-shaped asteroid 216 Kleopatra in its equatorial plane, assuming that its density is uniform. Note how the potential is continuous across its surface, denoted by the heavy contour. The scale bar is labeled in m^2/s^2.

POLY-HEDRON	-FACES-	-EDGES-	-VERTICES-	—FORM— FACTOR
TETRA-HEDRON	4	6	4	2.3017
CUBE	6	12	8	2.4453
OCTA-HEDRON	8	12	6	2.4575
DODECA-HEDRON	12	30	20	2.4982
ICOSA-HEDRON	20	30	12	2.5027
SPHERE	∞	∞	∞	2.51327

Table 1. Gravitational form factors F of homogeneous Platonic solids and spheres.

As a further comparison, FIGURE 3 contours the gravitational form factor F as a function of the aspect ratios b/a and c/b for ellipsoids with principal radii $a \geq b \geq c$, while FIGURE 4 does the same for cuboids with sides $a \geq b \geq c$. Note that for $b/a = c/b = 1$, Fig. 3 recovers the value $F \approx 2.513$ for a sphere, while Fig. 4 recovers the value $F \approx 2.445$ for a cube. But in either case, form factors of 2.05 like Kleopatra's require extreme aspect ratios on the order of 1/4, roughly consistent with Kleopatra's shape. This confirms that Kleopatra is weakly bound gravitationally because of its extreme elongation.

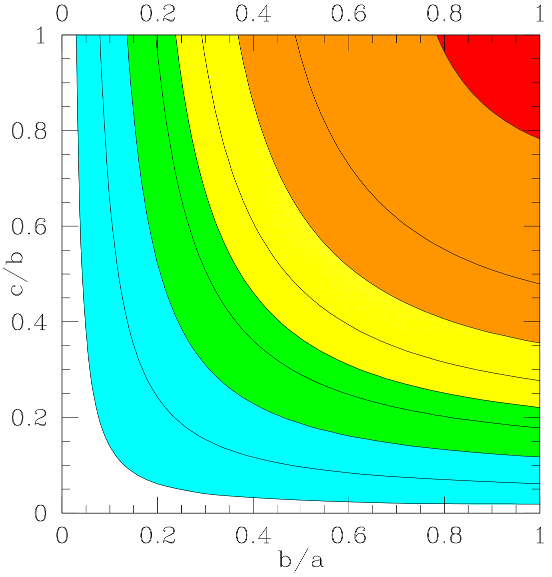

Figure 3: Gravitational form factor F for ellipsoids of principal radii $a \geq b \geq c$. Contour levels: 1.0, 1.5, 1.8, 2.0, 2.1, 2.2, 2.3, 2.4, and 2.5 .

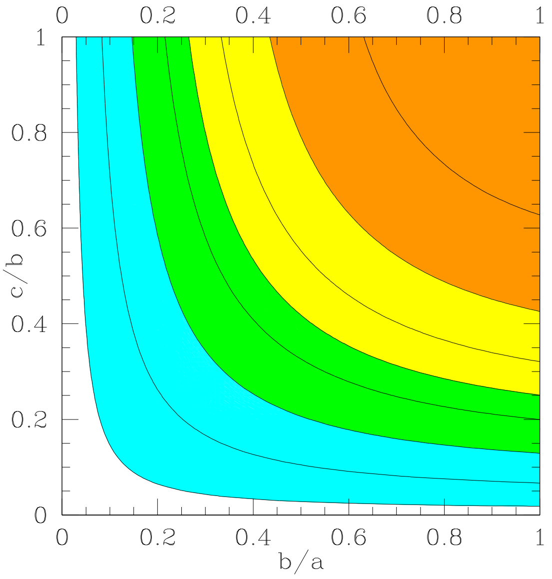

Figure 4: Gravitational form factor F for cuboids (rectangular parallelepipeds) of sides $a \geq b \geq c$, in the same format as Fig. 3. Contour levels: 1.0, 1.5, 1.8, 2.0, 2.1, 2.2, 2.3, and 2.4 .

ROTATION

The third contribution to a body's specific disruption energy Q_D^* is its specific energy of rotation $Q_R^* \equiv E_R/M$, where its kinetic energy of rotation E_R is just $I\omega^2/2$. Here ω is the object's spin angular speed and I is its moment of inertia about its axis of rotation. Note that the contribution of this term to the form factor F also scales as the square of the body's mean radius, but that its contributions to Q_D^* and F both are *negative*.

For example, Kleopatra rotates with a period of 5.385 hours and an angular speed $\omega = 3.241 \times 10^{-4}$ cycles/sec about its axis of least inertia, with $I \approx 4.78 \times 10^{27}$ kg m^2. Then $E_R \approx 2.51 \times 10^{20}$ Joules and $Q_R^* \approx 54$ m^2/s^2. This corresponds to a contribution of only –0.049 to the form factor F, leaving a total of $F \approx 2.00$.

ENERGY REBATE

Finally, in calculating the gravitational binding energy E_G, we have assumed that the body's entire mass M is dispersed to infinity. In fact, it suffices to re-arrange the body into two widely separated spheres, each with half of the mass M of the original object. This saves energy amounting to $G[9\pi\rho M^5]^{1/3}/5$ from E_G, corresponding to a savings of $G[9\pi\rho M^2]^{1/3}/5$ from Q_G^*, and equivalent to reducing the gravitational form factor F by a considerable amount $2\pi\sqrt[3]{2}/5 \approx 1.583$.

Thus the form factor of a non-rotating sphere would be reduced from 2.513 to 0.930, while that for Kleopatra would be reduced from $\sim$ 2.00 to only $\sim$ 0.62 . By means of this "energy rebate" and/or rotational energy, it is possible to reduce the gravitational form factor to zero.

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