Mass Extinctions as Lognormal Stochastic Processes

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ABSTRACT. In a recent paper (ref. [1]), this author investigated a mathematical model representing a Mass Extinction as a lognormal stochastic process in the number of Living Species suddenly undergoing a decrease. Cases of practical interest developed by the author analytically in detail were:

1) The K-Pg mass extinction ("end of dinosaurs", 65 million years ago) where the mean value decreased like an exponential ("Geometric Brownian Motion") or like the descending branch of a parabola.

2) The case when the mean value changed in time like a cubic (Markov-Korotayev model of Evolution).

The new, important mathematical feature presented in this paper is the case when the mean value curve of this lognormal process is ***arbitrary***, so that the model may be used to represent mass extinctions of any kind.

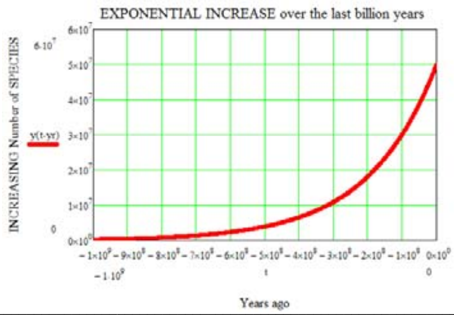
We extend the results in [10] for the benefit of simulating future mass extinctions that could be caused by the impact of an asteroid or a comet on Earth.

BASIC REFERENCE ABOUT MASS EXCTINTIONS AS DIVERSE REALIZATIONS IN TIME OF THE LOGNORMAL STOCHASTIC PROCESS :

[10] C. Maccone, "Evolution and Mass Extinctions as Lognormal Stochastic Processes", International Journal of Astrobiology, Vol. 13, issue 4, pages 290-309 (2014).

DARWINIAN EVOLUTION AS THE EXPONENTIAL INCREASE OF THE NUMBER OF LIVING SPECIES. Consider Darwinian Evolution. In order to cast it into a mathematically fruitful form, we may regard it as the exponentially increasing number of living species on Earth starting 3.5 billion years ago. In other words, 3.5 billion years ago there was on Earth only one living species (questionably RNA ?) whereas now there may be (say) 50 million living species or more Note that the actual number of species currently living on earth does not really matter as a number for us: we just want to stress the ***exponential*** character of the growth of species. Thus, we shall assume that the number of living species on Earth increases in time as $E(t) = A\,e^{B\,t}$ where *A* and *B* are two positive constants. A few steps show then that the numeric values of these two constants are, respectively :

$$A = 50 \text{ million species} = 5\cdot 10^{7} \text{ species} \qquad B = -\frac{\ln\left(5\cdot 10^{7}\right)}{-3.5\cdot 10^{9}\ \text{year}} = \frac{1.605\cdot 10^{-16}}{\text{sec}}.$$



***b*-LOGNORMALS AND LOGNORMALS.** Just for reference, we write here the equation of the lognormal probability density function starting at any positive time *b*>0, that we call b-lognormal in Maccone 2011 #2. The ordinary lognormal simply is the special case *b*=0 of the *b*-lognormal. The letter *b* stands for "birth instant".

$$\text{b\_lognormal}(t,\mu,\sigma,b) = \frac{1}{\sqrt{2\pi}\,\sigma\,(t-b)}\, e^{-\frac{(\ln(t-b)-\mu)^2}{2\sigma^2}} \qquad \text{holding for } t > b \text{ and up to } t = \infty.$$

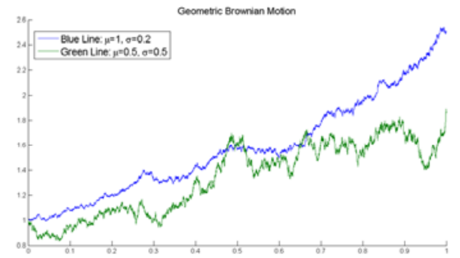
INTRODUCING THE "DARWIN" (d) UNIT, MEASURING THE AMOUNT OF EVOLUTION THAT A GIVEN SPECIES REACHED. In all sciences "to measure is to understand". In physics and chemistry this is done by virtue of units such as the meter, second, kilogram, coulomb, etcetera. So, it appears useful to introduce a new unit measuring the degree of evolution that a certain species has reached at a certain time *t* of Darwinian Evolution, and the obvious name for such a new unit is the "Darwin", denoted by a lower case "d". For instance, if we adopt the exponential evolution curve described in the previous section, we might say that the dominant species on Earth right now (Humans) have reached an evolution level of 50 million darwins. How many darwins may have an alien civilization already reached? Certainly more than 50 millions, i.e. more than 50 Md, but we will not check out until SETI succeeds for the first time. We are not going to discuss further this notion of measuring the "amount of evolution" since we are aware that endless discussions might come out of it. But it is clear to us that such a new measuring unit (and ways to measure it for different species) will sooner or later have to be introduced to make Evolution a fully quantitative science.

GEOMETRIC BROWNIAN MOTION (GBM) IS THE KEY TO STOCHASTIC EVOLUTION WITH AN INCREASING EXPONENTIAL TREND.

We now make a major step ahead: the number *N* in the Statistical Drake Equation, yielding the number of extraterrestrial civilizations now existing and communicating in the Galaxy, is replaced in this section by a ***stochastic process*** *N*(*t*), jumping up and down in time like the number *e* raised to a Brownian motion, but actually in such a way that ***its mean value keeps increasing exponentially in time as*** $\langle N(t)\rangle = N_0\, e^{\mu t}$. This evolution in time of is just what we expect to happen in the Galaxy, where the overall number of ET civilizations does probably ***increase*** (rather than decrease) in time because of the obvious technological evolution of each civilization. But this scenario is a stochastic one, rather than a deterministic one, and certainly does not exclude temporary setbacks, like the end of civilizations due to causes as diverse as: asteroid and comet impacts, rogue planets or stars, arriving from somewhere and disrupting the gravitational stability of the planetary system, supernova explosions that would "fry" entire nearby ET civilizations, ET nuclear wars, and possibly more causes of civilization end that we don't know about yet. The GMB pdf is the lognormal:

$$N(t)\_\text{pdf}\left(n, N_0, \mu_{GBM}, \sigma_{GBM}, t\right) = \frac{1}{\sqrt{2\pi}\,\sigma_{GBM}\,\sqrt{t}\,n}\, e^{-\frac{\left[\ln(n) - \left(\ln N_0 + \mu_{GBM}\, t - \frac{\sigma_{GBM}^2\, t}{2}\right)\right]^2}{2\sigma_{GBM}^2\, t}} \qquad \text{for} \quad 0 \le n \le \infty.$$

GBMs are of paramount importance in the mathematics of finance (Black-Sholes models). We have thus proven that the GBM used in the mathematics of finance is the same thing as the exponentially increasing process yielding the number of communicating ET civilizations in the Galaxy!



DARWINIAN EVOLUTION AS A GBM IN THE NUMBER OF LIVING SPECIES ON EARTH OVER THE LAST 3.5 BILLION YEARS. Having understood what GBMs are, it is now possible to re-define Darwinian Evolution, as it unfolded on Earth over the last 3.5 billion years, as just one single realization of a GBM in the number of living species on Earth over the last 3.5 billion years. All equations are the same, as for the process *N*(*t*), of course: only numbers change.

MASS EXTINCTION OF DINOSAURS 64 MILLION YEARS AGO AS A DECREASING LOGNORMAL STOCHASTIC PROCESS IN THE TIME OVER (SAY) 1000 AND 2000 YEARS AFTER THE ASTEROID IMPACT.

The following three plots are taken from Ref. [10] (2014) and show, from left to right: 1) the exponential decrease (i.e. a decreasing GBM) of the number of living species over 1000 years after the impact; 2) in the middle, the parabolic decrease of the number of living species over the same 1000 years; 3) on the right, the two above models are SUPERIMPOSED over 2000 years: we see that the parabolic model also takes into account the RECOVERY of life after the impact. In all plots, the SOLID, THICK MIDDLE CURVE is the mean value of the lognormal stochastic process, while the upper and lower THIN CURVES are the upper and lower STANDARD DEVIATION CURVES, around which the lognormal process is expected to unfold in time.

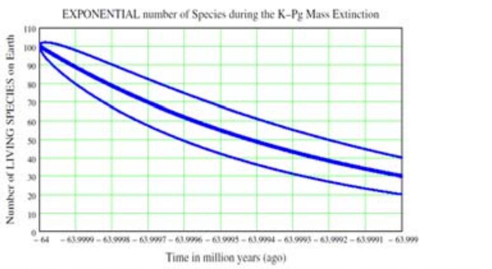


Fig. 3. The K–Pg mass extinction as a decreasing GBM in the number of living species over 1000 years after impact. The maximum of the upper standard deviation curve has the numeric value -6.399983×10^{7} years given by (41).

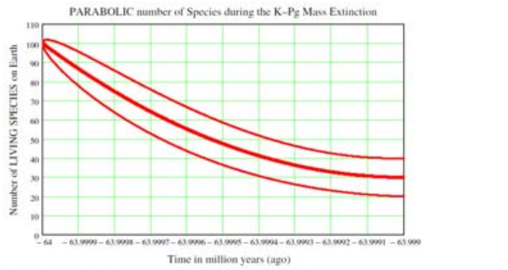


Fig. 4. The K–Pg mass extinction as a decreasing *parabola* in the number of living species over 1000 years after impact. The five numeric input values for this plot are just the same as those used for the construction of Fig. 3 in order to allow a perfect comparison between the two models, exponential (i.e. GBM-based) and parabolic.

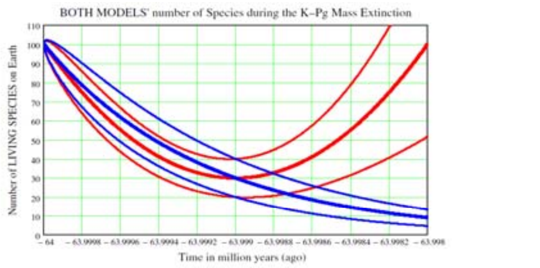


Fig. 6. If we *double* the horizontal axis time window of Fig. 5, then the result is the current Fig. 6. It clearly shows that the parabolic model (in red) allows for the recovery of life on Earth after the nuclear winter, while the GBM does not so. Thus, the parabolic lognormal process is a better model than the decreasing exponential (GBM) process.

MARKOV-KOROTAYEV (2007) MODEL OF BIODIVERSITY DURING THE PHANEROZOIC (542 million years) REWRITTEN BY THIS AUTHOR AS A LOGNORMAL PROCESS WITH A CUBIC MEAN VALUE (CUBIC TREND).

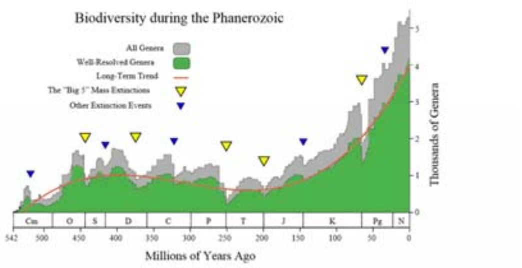


Fig. 7. During the Phanerozoic the biodiversity shows a steady but not monotonic increase from near zero to several thousands of genera.

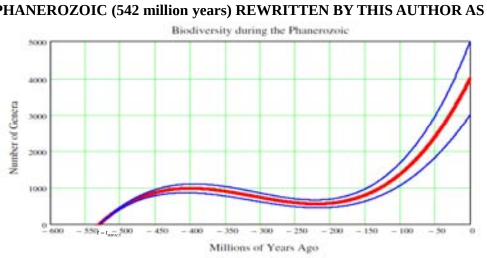


Fig. 8. Our cubic mean value curve (thick red solid curve) plus and minus the two standard deviation curves (thin solid blue curves) give more mathematical information than just the previous Fig. 7. In fact, we now have the two standard deviation curves of the lognormal stochastic process *L*(*t*) that are completely missing in the Markov–Korotayev theory and in their plot shown in Fig. 7. We thus claim that our cubic mathematical theory of the lognormal stochastic process *L*(*t*) is a 'more profound mathematization' than the Markov–Korotayev theory of evolution since it is stochastic, rather than just deterministic. This completes our 'stochastic extension' of the Markov–Korotayev evolution model.

Let our lognormal process start at *t*=*ts*=*t*Impact with value *Ns*=*N*Impact. Let its mean value be *Ne*=*N*End at the end-time *t*=*te*=tEnd, with a standard deviation of plus or minus δN_{End} above and below *Ne*. Then the decreasing GBM's mu and sigma are given by, respectively:

$$\mu = -\frac{\ln\left(\frac{N_{\text{Impact}}}{N_{\text{End}}}\right)}{t_{\text{End}} - t_{\text{Impact}}} \qquad \text{Time\_Lapse} = t_{\text{End}} - t_{\text{Impact}} \qquad \sigma = \sqrt{\frac{\ln\left[1 + \left(\frac{\delta N_{\text{End}}}{N_{\text{End}}}\right)^2\right]}{\text{Time\_Lapse}}}.$$

The upper (+) and lower (-) standard deviation curves have the equations :

$$\text{standard\_deviation\_curves}(t) = m(t)\left[1 \pm \sqrt{e^{\sigma^2 (t-ts)} - 1}\right].$$

Finally, the parabolic mean value and the cubic mean value have equations :

$$m_{\text{parabola}}(t) = (N_{\text{Impact}} - N_{\text{End}})\left[\frac{(t - t_{\text{Impact}})^2}{(t_{\text{End}} - t_{\text{Impact}})^2} - 2\frac{t - t_{\text{Impact}}}{t_{\text{End}} - t_{\text{Impact}}}\right] + N_{\text{Impact}}.$$

$$\text{Cubic}(t) = (Ne - Ns)\frac{(t - ts)\left[2(t-ts)^2 - 3(t_{\text{Max}} + t_{\text{min}} - 2ts)(t - ts) + 6(t_{\text{Max}} - ts)(t_{\text{min}} - ts)\right]}{(te - ts)\left[2(te - ts)^2 - 3(t_{\text{Max}} + t_{\text{min}} - 2ts)(te - ts) + 6(t_{\text{Max}} - ts)(t_{\text{min}} - ts)\right]} + Ns.$$

CONCLUSION. We have provided a new mathematical model (Evo-SETI Theory) capable of accounting for Darwinian Evolution as THE ONE PARTICULAR REALIZATION OF GEOMETRIC BROWNIAN MOTION in the number of living species on Earth that occurred over the last 3.5 billion years of life evolution. It will therefore be possible to SIMULATE by virtue of a numeric codes much of Darwininan Evolution and Cladistics as we know it as of 2015.

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